MEINARDUS: Calculations

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Inhaltsverzeichnis

0 Einleitung, Übersicht

Es werden Berechnungen von MEINARDUS dokumentiert. Die Berechnungen zu einer Funktion werden jeweils in einem eigenen Kapitel beschrieben. Konvention für die Kapitelüberschrift:

Approximationen zu f(x) = <function>

wobei "<function>" für eine Bezeichnung der jeweiligen Funktion steht.

Für jede Berechnung wird angegeben

- die Iterationsschritte:
  relevante Teile der jeweiligen MEINARDUS Log-Daten
- eine Auswertung der berechneten Näherungen mit EXPAP_EVAL:
  relevanten Date der EXPAP_EVAL Log-Daten

0.1 Vorgehensweise zur Erstellung dieses Dokuments

- Ausführung von MEINARDUS
- In Verzeichnis `Calculations/Logs` Erstellung einer TEX-Datei mit den relevanten Teilen der MEINARDUS Log-Datei
- In Verzeichnis `EXPAPP_EVAL/Jobs` Erstellung eines Unterverzeichnisses <subdir>.
  In diesem Unterverzeichnis Erstellung von `EXPAPP_EVAL` Job-Dateien
  "<subdir>-step<i>"
  für die Auswertung der in den jeweiligen Iterationsschritten berechneten Näherungen. Hierzu kann die von MEINARDUS erzeugte Job-Datei `eval` verwendet werden.
  `EXPAPP_EVAL` mit jeder diese Job-Dateien ausführen.
- In Verzeichnis `Calculations/Evaluations` Erstellung eines Unterverzeichnisses <subdir>.
  In diesem Unterverzeichnis für jede `EXPAPP_EVAL` Job-Datei eine TEX-Datei erstellen mit den relevanten Teilen der `EXPAPP_EVAL` Log-Datei.
- Erstellen des entsprechenden Kapitels in einer `Calc-*.tex` Datei.

1 $\mathbf{V_3^0}$ - Approximationen zu $\mathbf{f(x) = \dfrac{1}{1+x}}$

Die Berechnungen von Beispiel 5.5 von [3], sie entsprechen den Berechnungen der Startfunktionen in §7.4 der Diplomarbeit [2]. (Es gibt kein “Beispiel 5.5” in [2].)

Anmerkung:
Die Berechnung von Abschnitt 1.2 ist identisch mit der Beispielrechnung der RZ-Programmdokumentation [1], p. 27-28 (“Ermittlung einer Approximation nach Meinardus”).

1.1 Die Berechnung: $\mathbf{I = [0, 1],\ x_0 = 0.0,\ x_3 = 1.0}$

1.1.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.5/Beispiel5.5a":
-------------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval                        : [0.00,1.00]
- Interval Equidistant Points     : [0.00,1.00]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------

***************************************************
*          The MEINARDUS approximation:           *
*                 a[i]                 t[i]       *
*         ------------------   ------------------ *
* i= 1:   +0.546271421802803   -0.277576311531820 *
* i= 2:   +0.401383367280923   -1.546985019769484 *
* i= 3:   +0.052345210916276   -4.336157075160293 *
*                                                 *
***************************************************
```

1.1.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.5/eval-a":
-------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                 t[i]
  i= 1:   +0.546271421802803   -0.277576311531820
  i= 2:   +0.401383367280923   -1.546985019769484
  i= 3:   +0.052345210916276   -4.336157075160293
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
              x[i]                    y[i]
        ------------------   ------------------
i= 0:   +0.058419741719676   -0.000010725167211
i= 1:   +0.274946760913293   +0.000001768165096
i= 2:   +0.489553937155876   -0.000000716972688
i= 3:   +0.705085328019672   +0.000000622213902
i= 4:   +0.924943684579506   -0.000001289514225

Norm of error function:       0.000010725167211
Relative deviation:           0.941985622253863

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
              x[i]
        ------------------
i= 0:   +0.200000000084656
i= 1:   +0.399999999589713
i= 2:   +0.600000000862734
i= 3:   +0.799999999183648
```

1.2 Die Berechnung: $\mathbf{I} = [\mathbf{0}, \mathbf{1}],\ \mathbf{x_0} = \mathbf{0.1},\ \mathbf{x_3} = \mathbf{0.9}$

1.2.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.5/Beispiel5.5b":
-------------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval                      : [0.00,1.00]
- Interval Equidistant Points   : [0.10,0.90]
- No of Root Points             : 100
- No of Plot Points             : 100
- Plot-Indicator                : 1
- output                        : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                 t[i]       *
*         ------------------   ------------------ *
* i= 1:   +0.545644214548501   -0.277434496518055 *
* i= 2:   +0.400482842986620   -1.540603343594550 *
* i= 3:   +0.053824480225551   -4.282493872762781 *
*                                                 *
***************************************************
```

1.2.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.5/eval-b":
-------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
               a[i]                 t[i]
  i= 1:   +0.545644214548501   -0.277434496518055
  i= 2:   +0.400482842986620   -1.540603343594550
  i= 3:   +0.053824480225551   -4.282493872762781
```

```
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   +0.000048462239328
i= 1:   +0.148285553072205   -0.000002124561810
i= 2:   +0.321682262482468   +0.000000400652767
i= 3:   +0.493339405524613   -0.000000182174950
i= 4:   +0.665600173247601   +0.000000174872733
i= 5:   +0.841160198596452   -0.000000397018970
i= 6:   +1.000000000000000   +0.000004243995534

Norm of error function:       0.000048462239328
Relative deviation:           0.996391567222378

----------------------------------------------------


The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +0.100000000000441
i= 1:   +0.259999999566054
i= 2:   +0.420000002262248
i= 3:   +0.579999995552120
i= 4:   +0.740000003812111
i= 5:   +0.899999998779755
```

2 $\mathbf{V_2^0}$ - Approximationen zu $\mathbf{f(x) = \sqrt{x}}$

Die Berechnungen von Beispiel 5.2 von §5.4 der Diplomarbeit [2].

2.1 Berechnung: $\mathbf{x_0 = +0.01}$ $\mathbf{x_3 = +0.8}$

2.1.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.2/ex52-1-terse":
-------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                        : [0.00,1.00]
- Interval Equidistant Points     : [0.01,0.80]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 0
- output                          : terse

-----------  End Of Initialization  ----------


***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                  t[i]      *
*          ------------------   ------------------ *
* i= 1:   +0.558037442886304   +0.606012117131778 *
* i= 2:   -0.483373717342265   -4.646084908083401 *
*                                                 *
***************************************************
```

2.1.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../../MEINARDUS/Jobs/eval":
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.558037442886304   +0.606012117131778
  i= 2:   -0.483373717342265   -4.646084908083401
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.074663725544039
i= 1:   +0.059065155596638   +0.032029082452410
i= 2:   +0.377629910366786   -0.003400945312621
i= 3:   +0.681148056382007   +0.002523908487339
i= 4:   +1.000000000000000   -0.018302155985205

Norm of error function:       0.074663725544039
Relative deviation:           0.966196322659388

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +0.010000000000000
i= 1:   +0.273333333333337
i= 2:   +0.536666666666661
i= 3:   +0.800000000000007
```

2.2 Berechnung: $x_0 = +0.1$ $x_3 = +0.9$

2.2.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.2/ex52-2-terse":
-------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Interval Equidistant Points    : [0.10,0.90]
- No of Root Points              : 50
- No of Plot Points              : 100
- Plot-Indicator                 : 0
- output                         : terse

-----------  End Of Initialization  ----------


***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                 t[i]       *
*         ------------------   ------------------ *
* i= 1:   +0.655627087722369   +0.450327954645726 *
* i= 2:   -0.496967378732112   -2.961059561377337 *
*                                                 *
***************************************************
```

2.2.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../../MEINARDUS/Jobs/eval":
--------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                 t[i]
  i= 1:   +0.655627087722369   +0.450327954645726
  i= 2:   -0.496967378732112   -2.961059561377337
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
              x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.158659708990257
i= 1:   +0.174296383570430   +0.004938367824643
i= 2:   +0.480229479591281   -0.001041228974349
i= 3:   +0.784261915135921   +0.000976987512598
i= 4:   +1.000000000000000   -0.002840178514881

Norm of error function:       0.158659708990257
Relative deviation:           0.993842245653821

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
              x[i]
        ------------------
i= 0:   +0.100000000000001
i= 1:   +0.366666666666642
i= 2:   +0.633333333333347
i= 3:   +0.899999999999992
```

3 $\mathbf{V_2^0}$ - Approximationen zu $\mathbf{f(x) = 2 + x \mid 2 - x}$

Die Berechnungen von Beispiel 5.3 von §5.4 der Diplomarbeit [2]:

- Abschnitte 3.1 bis 3.8 sind Beispiele für die Nichtdurchführbarkeit des Algorithmus von Meinardus: Polynom Q hat komplexe Nullstellen.
- Abschnitte 3.9 bis 3.11 berechnen Näherungen. Die Funktion f(x) ist (an der Stelle $x = 0$) nicht differenzierbar, Auswertungen mit EXPAPP_EVAL entfallen daher.

3.1 Berechnung: $\mathbf{x_0 = -1.0 \; x_3 = +1.0}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-1":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-1.00,1.00]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.6000000000E+00
q[2] = +1.0000000000E+00

2 complex roots: +0.800000  +i*0.600000
                 +0.800000  -i*0.600000
```

3.2 Berechnung: $\mathbf{x_0 = -0.9}$ $\mathbf{x_3 = +0.9}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-2":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.90,0.90]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.6470588235E+00
q[2] = +1.0000000000E+00

2 complex roots: +0.823529  +i*0.567274
                 +0.823529  -i*0.567274
```

3.3 Berechnung: $\mathbf{x_0 = -0.8}$ $\mathbf{x_3 = +0.8}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-3":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                          : [-1.00,1.00]
- Interval Equidistant Points       : [-0.80,0.80]
- No of Root Points                 : 50
- No of Plot Points                 : 100
- Plot-Indicator                    : 1
- output                            : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.6923076923E+00
q[2] = +1.0000000000E+00

2 complex roots: +0.846154  +i*0.532939
                 +0.846154  -i*0.532939
```

3.4 Berechnung: $x_0 = -0.7$ $x_3 = +0.7$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-4":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.70,0.70]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.7358490566E+00
q[2] = +1.0000000000E+00

2 complex roots: +0.867925  +i*0.496696
                 +0.867925  -i*0.496696
```

3.5 Berechnung: $x_0 = -0.5$ $x_3 = +0.5$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-5":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.50,0.50]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.8181818182E+00
q[2] = +1.0000000000E+00

2 complex roots: +0.909091  +i*0.416598
                 +0.909091  -i*0.416598
```

3.6 Berechnung: $x_0 = -0.8$ $x_3 = +0.7$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-6":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.80,0.70]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -2.0547945205E+00
q[2] = +1.4109589041E+00

2 complex roots: +0.728155  +i*0.422525
                 +0.728155  -i*0.422525
```

3.7 Berechnung: $x_0 = -0.7$ $x_3 = +0.8$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-7":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                       : [-1.00,1.00]
- Interval Equidistant Points    : [-0.70,0.80]
- No of Root Points              : 50
- No of Plot Points              : 100
- Plot-Indicator                 : 1
- output                         : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -1.4563106796E+00
q[2] = +7.0873786408E-01

2 complex roots: +1.027397  +i*0.596166
                 +1.027397  -i*0.596166
```

3.8 Berechnung: $\mathbf{x_0 = -0.5\ x_3 = +0.4}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-failure-8":
----------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.50,0.40]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

The coefficients of polynomial Q
degree of Q is 2
q[0] = +1.0000000000E+00
q[1] = -2.6153846154E+00
q[2] = +1.8717948718E+00

2 complex roots: +0.698630  +i*0.214855
                 +0.698630  -i*0.214855
```

3.9 Die Berechnung: $\mathbf{x_0 = -0.5\ x_3 = +0.35}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-1":
---------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                         : [-1.00,1.00]
- Interval Equidistant Points      : [-0.50,0.35]
- No of Root Points                : 50
- No of Plot Points                : 100
- Plot-Indicator                   : 1
- output                           : verbose

-----------  End Of Initialization  ----------

***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                t[i]        *
*         ------------------   ------------------ *
* i= 1:   -0.898915427649777   +2.705946469619257 *
* i= 2:   +2.820549263162334   +0.974913358890773 *
*                                                 *
***************************************************
```

3.10 Die Berechnung: $\mathbf{x_0 = -0.5\ x_3 = +0.3}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-2":
---------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                        : [-1.00,1.00]
- Interval Equidistant Points     : [-0.50,0.30]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : verbose

-----------  End Of Initialization  ----------

***************************************************
*          The MEINARDUS approximation:           *
*                a[i]                 t[i]        *
*        ------------------   ------------------ *
* i= 1:   -0.185691048863594   +5.381257438718608 *
* i= 2:   +2.138865180422977   +0.692894767899260 *
*                                                 *
***************************************************
```

3.11 Die Berechnung: $\mathbf{x_0 = -0.5\ x_3 = +0.25}$

```
Input from job file "../Jobs/Beispiel5.3/Beispiel5.3-3":
---------------------------------------------------------------
- Function: Funktion von Beispiel 5.3
- Approximation with respect to V_2
- Interval                      : [-1.00,1.00]
- Interval Equidistant Points   : [-0.50,0.25]
- No of Root Points             : 50
- No of Plot Points             : 100
- Plot-Indicator                : 1
- output                        : verbose

-----------  End Of Initialization  ----------

***************************************************
*          The MEINARDUS approximation:           *
*                a[i]                   t[i]      *
*         ------------------   ------------------ *
* i= 1:   -0.050428833997798  +10.207441475082318 *
* i= 2:   +2.050428833997866   +0.624759329326523 *
*                                                 *
***************************************************
```

4 Approximationen für die Riemannsche Zeta-Funktion

Die Berechnungen von Beispiel 5.4 von [3], sie entsprechen den Berechnungen der Startfunktionen in §8 der Diplomarbeit [2]. (Es gibt kein "Beispiel 5.4" in [2].)

4.1 Die Berechnung für $\mathbf{V_1^0}$, $\mathbf{I = [2, 3]}$ mit $\mathbf{x_0 = 2.333}$ $\mathbf{x_1 = 2.666}$

4.1.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-1a":
------------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_1
- Interval                        : [2.000,3.000]
- Interval Equidistant Points     : [2.333,2.666]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------



******************************************************
*            The MEINARDUS approximation:            *
*                  a[i]                  t[i]        *
*          ------------------   ------------------  *
* i= 1:   +2.793905734833698   -0.291505518304489  *
*                                                    *
******************************************************
```

4.1.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-1a":
  --------------------------------------------------------
  - Function: Riemannsche Zeta-Funktion
  - Approximation with respect to V_1
  - Interval I                    : [2.00,3.00]
  - Distance of equidistant points: 0.0100
  - The parameters of approximation:
                  a[i]                 t[i]
    i= 1:   +2.793905734833698   -0.291505518304489
  - output: terse
  ----------------------------------------------------

  The local extrema of f-E(a) in interval I:
  ------------------------------------------
               x[i]                 y[i]
          ------------------   ------------------
  i= 0:   +2.000000000000000   +0.085333899843164
  i= 1:   +2.488132556394935   -0.006620062500357
  i= 2:   +3.000000000000000   +0.036820524105029

  Norm of error function:       0.085333899843164
  Relative deviation:           0.922421657600044

  ----------------------------------------------------

  The exact zeroes of f-E(a) in interval I:
  ---------------------------------------
               x[i]
          ------------------
  i= 0:   +2.333000000000005
  i= 1:   +2.665999999999999
```

4.2 Die Berechnung für $\mathbf{V_1^0}$, $\mathbf{I = [2, 4]}$ mit $\mathbf{x_0 = 2.666, x_1 = 3.333}$

4.2.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-1b":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_1
- Interval                        : [2.000,4.000]
- Interval Equidistant Points     : [2.666,3.333]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------


***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                t[i]        *
*         ------------------   ------------------ *
* i= 1:   +2.015963567988093   -0.169096199764800 *
*                                                 *
***************************************************
```

4.2.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-1b":
------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_1
- Interval I                    : [2.00,4.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                t[i]
  i= 1:   +2.015963567988093   -0.169096199764800
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.207434950379000
i= 1:   +2.966254980565173   -0.011928701695711
i= 2:   +4.000000000000000   +0.057302869877868

Norm of error function:       0.207434950379000
Relative deviation:           0.942494253384418

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.665999999999987
i= 1:   +3.333000000000030
```

4.3 Die Berechnung für $\mathbf{V_2^0},\ \mathbf{I = [2, 3]}$ mit $\mathbf{x_0 = +2.200\ x_3 = +2.800}$

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-2a":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_2
- Interval                       : [2.000,3.000]
- Interval Equidistant Points    : [2.200,2.800]
- No of Root Points              : 50
- No of Plot Points              : 100
- Plot-Indicator                 : 1
- output                         : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                 t[i]       *
*         ------------------   ------------------ *
* i= 1:   +1.525165278411235   -0.092521466386125 *
* i= 2:  +24.876184782939173   -2.097844336290357 *
*                                                 *
***************************************************
```

4.3.1 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-2a":
--------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_2
- Interval I                    : [2.00,3.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                  t[i]
  i= 1:   +1.525165278411235   -0.092521466386125
  i= 2:  +24.876184782939173   -2.097844336290357
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.002772611534855
i= 1:   +2.269890546842699   -0.000070165587340
i= 2:   +2.493048866342793   +0.000027460759692
i= 3:   +2.717689145509421   -0.000034992325938
i= 4:   +3.000000000000000   +0.000574227704832

Norm of error function:       0.002772611534855
Relative deviation:           0.990095706035031

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.200000000000484
i= 1:   +2.399999999997615
i= 2:   +2.600000000003557
i= 3:   +2.799999999998146
```

4.4 Die Berechnung für $\mathbf{V_2^0}$, $\mathbf{I = [2, 4]}$ mit $\mathbf{x_0 = +2.400}$ $\mathbf{x_3 = +3.600}$

4.4.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-2b":
-----------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_2
- Interval                        : [2.000,4.000]
- Interval Equidistant Points     : [2.400,3.600]
- No of Root Points               : 50
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------



***************************************************
*          The MEINARDUS approximation:           *
*                a[i]                t[i]         *
*        ------------------   ------------------ *
* i= 1:   +1.262562853686527   -0.043145667715900 *
* i= 2:  +11.959298968418540   -1.620042255997213 *
*                                                 *
***************************************************
```

4.4.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-2b":
--------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_2
- Interval I                    : [2.00,4.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                t[i]
  i= 1:   +1.262562853686527   -0.043145667715900
  i= 2:  +11.959298968418540   -1.620042255997213
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
              x[i]                y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.018417995847323
i= 1:   +2.532334924421203   -0.000329476076090
i= 2:   +2.978724925416265   +0.000104601981427
i= 3:   +3.429416092467418   -0.000112486632801
i= 4:   +4.000000000000000   +0.001550634605656

Norm of error function:        0.018417995847323
Relative deviation:            0.994320664295186

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
              x[i]
        ------------------
i= 0:   +2.400000000000084
i= 1:   +2.799999999999168
i= 2:   +3.200000000002207
i= 3:   +3.599999999998908
```

4.5 Die Berechnung bzgl. $\mathbf{V_3^0}$, $\mathbf{I = [2,3]}$ mit $\mathbf{x_0 = +2.050}$ $\mathbf{x_5 = +2.850}$

4.5.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-3a":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_3
- Interval                        : [2.000,3.000]
- Interval Equidistant Points     : [2.050,2.850]
- No of Root Points               : 100
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +1.236018240085825   -0.039532424048109 *
* i= 2:   +8.421439207750364   -1.466241490358296 *
* i= 3: +301.455919776439089   -4.310972926757475 *
*                                                 *
***************************************************
```

4.5.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-3a":
-----------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_3
- Interval I                    : [2.00,3.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                 t[i]
  i= 1:   +1.236018240085825   -0.039532424048109
  i= 2:   +8.421439207750364   -1.466241490358296
  i= 3: +301.455919776439089   -4.310972926757475
- output: terse
-------------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.000018120188707
i= 1:   +2.098312125875493   -0.000003209710625
i= 2:   +2.271753088341870   +0.000000607596905
i= 3:   +2.443449367758479   -0.000000277927266
i= 4:   +2.615736632471019   +0.000000268855807
i= 5:   +2.791298182362147   -0.000000616108733
i= 6:   +3.000000000000000   +0.000014744230119

Norm of error function:       0.000018120188707
Relative deviation:           0.985162637600810

-------------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.050000000025224
i= 1:   +2.209999999860452
i= 2:   +2.370000000396446
i= 3:   +2.529999999593800
```

```
i= 4:   +2.690000000328389
i= 5:   +2.849999999958194
```

4.6 Die Berechnung bzgl. $\mathbf{V_3^0}$, $\mathbf{I = [2, 4]}$ mit $\mathbf{x_0 = +2.050}$ $\mathbf{x_5 = +3.700}$

4.6.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-3b":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_3
- Interval                       : [2.000,4.000]
- Interval Equidistant Points    : [2.050,3.700]
- No of Root Points              : 50
- No of Plot Points              : 100
- Plot-Indicator                 : 1
- output                         : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                 a[i]                  t[i]      *
*         ------------------   ------------------ *
* i= 1:   +1.112198403902258   -0.017032105666695 *
* i= 2:   +5.076875370718804   -1.191695146776540 *
* i= 3: +113.053478781429916   -3.507624270930476 *
*                                                 *
***************************************************
```

4.6.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-3b":
--------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_3
- Interval I                    : [2.00,4.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                t[i]
  i= 1:   +1.112198403902258   -0.017032105666695
  i= 2:   +5.076875370718804   -1.191695146776540
  i= 3: +113.053478781429916   -3.507624270930476
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.000174994054377
i= 1:   +2.141270454026952   -0.000080012670744
i= 2:   +2.498315954553322   +0.000010598692075
i= 3:   +2.852750864957368   -0.000003617429192
i= 4:   +3.209056732452057   +0.000002723610793
i= 5:   +3.572898279609959   -0.000005002901153
i= 6:   +4.000000000000000   +0.000089811280987

Norm of error function:       0.000174994054377
Relative deviation:           0.984435980968494

----------------------------------------------------

The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.050000000000258
i= 1:   +2.379999999990608
i= 2:   +2.710000000045419
i= 3:   +3.039999999897667
```

```
i= 4:   +3.370000000092410
i= 5:   +3.699999999955332
```

4.7 Die Berechnung bzgl. V_4^0, $I = [2, 3]$ mit $x_0 = +2.100$ $x_7 = +2.800$

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-4a":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_4
- Interval                        : [2.000,3.000]
- Interval Equidistant Points     : [2.100,2.800]
- No of Root Points               : 100
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                  a[i]                  t[i]      *
*         ------------------   ------------------ *
* i= 1:   +1.108917301471468   -0.016846093164574 *
* i= 2:   +4.516500699859254   -1.151831800606335 *
* i= 3:  +56.338107956927068   -3.097712958237463 *
* i= 4: +3183.222936114931144   -6.533155909574688 *
*                                                 *
***************************************************
```

4.7.1 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-4a":
--------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_4
- Interval I                    : [2.00,3.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                  t[i]
  i= 1:   +1.108917301471468   -0.016846093164574
  i= 2:   +4.516500699859254   -1.151831800606335
  i= 3:  +56.338107956927068   -3.097712958237463
  i= 4: +3183.222936114931144   -6.533155909574688
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.000001057523388
i= 1:   +2.128519415323400   -0.000000010526563
i= 2:   +2.235636512555164   +0.000000001474830
i= 3:   +2.341335162461184   -0.000000000450611
i= 4:   +2.446692297434736   +0.000000000247354
i= 5:   +2.552223543216341   -0.000000000234028
i= 6:   +2.658469290619220   +0.000000000395019
i= 7:   +2.766736330976283   -0.000000001435239
i= 8:   +3.000000000000000   +0.000000433768881

Norm of error function:       0.000001057523388
Relative deviation:           0.999778701854386

----------------------------------------------------
```

```
The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.099999949494068
i= 1:   +2.200000875666556
i= 2:   +2.299994772684927
i= 3:   +2.400015736977787
i= 4:   +2.499973416324910
i= 5:   +2.600025719956488
i= 6:   +2.699986678312459
i= 7:   +2.800002874078424
```

4.8 Die Berechnung bzgl. $\mathbf{V_4^0}$, $\mathbf{I = [2,4]}$ mit $\mathbf{x_0 = +2.050}$ $\mathbf{x_7 = +3.600}$

4.8.1 Die Berechnung

```
Input from job file "../Jobs/Beispiel5.4/Beispiel5.4-4b":
---------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_4
- Interval                        : [2.000,4.000]
- Interval Equidistant Points     : [2.050,3.600]
- No of Root Points               : 100
- No of Plot Points               : 100
- Plot-Indicator                  : 1
- output                          : terse

-----------  End Of Initialization  ----------



***************************************************
*           The MEINARDUS approximation:          *
*                  a[i]                t[i]       *
*          ------------------   ------------------ *
* i= 1:   +1.048520425314867   -0.006758602512678 *
* i= 2:   +3.067446915963222   -0.980260258943750 *
* i= 3:  +27.094443001822096   -2.561189494307341 *
* i= 4: +928.206585131917791   -5.450993513150297 *
*                                                 *
***************************************************
```

4.8.2 Auswertungen mit EXPAPP_EVAL

```
Input from job file "../Jobs/Beispiel5.4/eval-5.4-4b":
--------------------------------------------------------
- Function: Riemannsche Zeta-Funktion
- Approximation with respect to V_4
- Interval I                    : [2.00,4.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                  t[i]
  i= 1:   +1.048520425314867   -0.006758602512678
  i= 2:   +3.067446915963222   -0.980260258943750
  i= 3:  +27.094443001822096   -2.561189494307341
  i= 4: +928.206585131917791   -5.450993513150297
- output: terse
----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
              x[i]                  y[i]
        ------------------   ------------------
i= 0:   +2.000000000000000   +0.000009147819680
i= 1:   +2.108048302672329   -0.000002041005539
i= 2:   +2.344431099450219   +0.000000198384914
i= 3:   +2.578277979090344   -0.000000044016324
i= 4:   +2.811714358692546   +0.000000018169084
i= 5:   +3.045792459703385   -0.000000013240497
i= 6:   +3.281747912616982   +0.000000017640502
i= 7:   +3.522595870245041   -0.000000051289565
i= 8:   +4.000000000000000   +0.000007688057560

Norm of error function:       0.000009147819680
Relative deviation:           0.998552606226839

----------------------------------------------------
```

```
The exact zeroes of f-E(a) in interval I:
---------------------------------------
               x[i]
        ------------------
i= 0:   +2.050000000001570
i= 1:   +2.271428567978464
i= 2:   +2.492857179146048
i= 3:   +2.714285560051560
i= 4:   +2.935714630066141
i= 5:   +3.157142438052369
i= 6:   +3.378571690848957
i= 7:   +3.599999933431924
```

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