# BRAESS:
# Calculations

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# Inhaltsverzeichnis

# Einleitung, Übersicht

Es werden Berechnungen von BRAESS dokumentiert. Die Berechnungen zu einer Funktion werden jeweils in einem eigenen Kapitel beschrieben.

Konvention für die Kapitelüberschrift:

*Approximationen zu f(x) = <function>*

wobei "<function>" für eine Bezeichnung der jeweiligen Funktion steht.

Für jede Berechnung werden in einem Abschnitt "Die Berechnung ..." angegeben

- die Iterationsschritte:
  relevante Teile der jeweiligen BRAESS Log-Daten
- eine Auswertung der berechneten Näherungen mit EXPAP_EVAL:
  relevanten Date der EXPAP_EVAL Log-Daten

# 1 Approximationen zu $\mathbf{f(x) = \sqrt{x}}$ bzgl. $\mathbf{V_1^0}$

Berechnungen zu $f(x) = \sqrt{x}$ mit unterschiedlichen Werten für $\delta_B$:

- $\delta_R = 0.01$ $\delta_B = 0.01$:
  "best approximation" berechnet mit "iteration step 4", s Abschnitt 1.1.
- $\delta_R = 0.01$ $\delta_B = 0.001$:
  "best approximation" berechnet mit "iteration step 4", s Abschnitt 1.2.
- $\delta_R = 0.001$ $\delta_B = 0.001$:
  "best approximation" berechnet mit "iteration step 4", s Abschnitt 1.3.
- $\delta_R = 0.001$ $\delta_B = 0.0001$:
  "best approximation" berechnet mit "iteration step 5", s Abschnitt 1.4.

Die ersten drei Berechnungen führen zum gleichen Ergebnis, die Berechnung mit $\delta_R = 0.001$ $\delta_B = 0.0001$ liefert eine bessere Approximation, s. Abschnitt 1.6.
Zum Vergleich werden in Abschnitt 1.5 Auswertungen der Berechnungen von §7.1 in [1], p. 118 ff, angegeben. Die Berechnung mit $\delta_R = 0.001$ $\delta_B = 0.0001$ entspricht der Berechnung von [1], §7.1A, s. Abschnitt 1.6.

## 1.1 Berechnung mit $\delta_\mathbf{R} = \mathbf{0.01}$, $\delta_\mathbf{B} = \mathbf{0.01}$

### 1.1.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt71/abschnitt71-terse-delta_r-2_b-2":
------------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                  t[i]
  i= 1:   +0.500000000000000   +0.000000000000000

------------  End Of Initialization  -----------
```

```
BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.125000000000000   +1.999999999999999 *
*************************************************

BRAESS - iteration step 2
The parameters r[i] of linear approximation:
r[0]=  +0.083637215741156
r[1]=  -0.360527646548126
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208637215741156   +1.639472353451873 *
*************************************************

BRAESS - iteration step 3
The parameters r[i] of linear approximation:
r[0]=  +0.001025418411059
r[1]=  +0.120360213961758
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209662634152215   +1.759832567413631 *
*************************************************
```

```
BRAESS - iteration step 4
The parameters r[i] of linear approximation:
r[0]=  +0.000289021688079
r[1]=  -0.008350707323088
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                   a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209951655840295   +1.751481860090543 *
*****************************************************

               END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

### 1.1.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4**

```
Input from job file "../Jobs/para71/para71-1-step4":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.209951655840295   +1.751481860090543
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.209951655840295
i= 1:   +0.421835941697434   +0.209954888798827
i= 2:   +1.000000000000000   -0.209980053835259

Norm of error function:       0.209980053835259
Relative deviation:           0.000135241392912

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.053088895609283
i= 1:   +0.842123198024851
```

## 1.2 Berechnung mit $\delta_{\mathbf{R}} = 0.01$, $\delta_{\mathbf{B}} = 0.001$

### 1.2.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt71/abschnitt71-terse-delta_r-2_b-3":
------------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-03
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                t[i]
  i= 1:   +0.500000000000000   +0.000000000000000

------------  End Of Initialization  -----------

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.125000000000000   +1.999999999999999 *
*                                                 *
*****************************************************

BRAESS - iteration step 2
The parameters r[i] of linear approximation:
r[0]=  +0.083637215741156
r[1]=  -0.360527646548126
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208637215741156   +1.639472353451873 *
*****************************************************
```

```
BRAESS - iteration step 3
The parameters r[i] of linear approximation:
r[0]=  +0.001025418411059
r[1]=  +0.120360213961758
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209662634152215   +1.759832567413631 *
*****************************************************

BRAESS - iteration step 4
The parameters r[i] of linear approximation:
r[0]=  +0.000289021688079
r[1]=  -0.008350707323088
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209951655840295   +1.751481860090543 *
*****************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-03
```

### 1.2.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4**

```
Input from job file "../Jobs/para71/para71-2-step4":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.209951655840295   +1.751481860090543
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.209951655840295
i= 1:   +0.421835941697434   +0.209954888798827
i= 2:   +1.000000000000000   -0.209980053835259

Norm of error function:       0.209980053835259
Relative deviation:           0.000135241392912

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.053088895609283
i= 1:   +0.842123198024851
```

## 1.3 Berechnung mit $\delta_{\mathbf{R}} = 0.001$, $\delta_{\mathbf{B}} = 0.001$

### 1.3.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt71/abschnitt71-terse-delta_r-3_b-3":
------------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-03
- Remez  termination criterion  : 1.000000E-03
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.500000000000000   +0.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 1
The parameters r[i] of linear approximation:
r[0]=  -0.375000000000000
r[1]=  +1.999999999999999
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.125000000000000   +1.999999999999999 *
*****************************************************

BRAESS - iteration step 2
The parameters r[i] of linear approximation:
r[0]=  +0.083637215741156
r[1]=  -0.360527646548126
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208637215741156   +1.639472353451873 *
*****************************************************
```

```
BRAESS - iteration step 3
The parameters r[i] of linear approximation:
r[0]=  +0.001025418411059
r[1]=  +0.120360213961758
Factor C=1.000000 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209662634152215   +1.759832567413631 *
**************************************************

BRAESS - iteration step 4
The parameters r[i] of linear approximation:
r[0]=  +0.000289021688079
r[1]=  -0.008350707323088
Factor C=1.000000 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209951655840295   +1.751481860090543 *
**************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-03
```

### 1.3.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4**

```
Input from job file "../Jobs/para71/para71-3-step4":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.209951655840295   +1.751481860090543
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.209951655840295
i= 1:   +0.421835941697434   +0.209954888798827
i= 2:   +1.000000000000000   -0.209980053835259

Norm of error function:       0.209980053835259
Relative deviation:           0.000135241392912

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.053088895609283
i= 1:   +0.842123198024851
```

## 1.4 Berechnung mit $\delta_{\mathbf{R}} = 0.001$, $\delta_{\mathbf{B}} = 0.0001$

### 1.4.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt71/abschnitt71-terse-delta_r-3_b-4":
------------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-04
- Remez  termination criterion  : 1.000000E-03
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.500000000000000   +0.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 1
The parameters r[i] of linear approximation:
r[0]=  -0.375000000000000
r[1]=  +1.999999999999999
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.125000000000000   +1.999999999999999 *
*****************************************************

BRAESS - iteration step 2
The parameters r[i] of linear approximation:
r[0]=  +0.083637215741156
r[1]=  -0.360527646548126
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208637215741156   +1.639472353451873 *
*****************************************************
```

```
BRAESS - iteration step 3
The parameters r[i] of linear approximation:
r[0]=  +0.001025418411059
r[1]=  +0.120360213961758
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209662634152215   +1.759832567413631 *
*************************************************

BRAESS - iteration step 4
The parameters r[i] of linear approximation:
r[0]=  +0.000289021688079
r[1]=  -0.008350707323088
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209951655840295   +1.751481860090543 *
*************************************************

BRAESS - iteration step 5
The parameters r[i] of linear approximation:
r[0]=  +0.000001583329108
r[1]=  -0.000029702644716
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209953239169403   +1.751452157445828 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
            with delta_B = 1.000000E-04
```

### 1.4.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 5**

```
Input from job file "../Jobs/para71/para71-4-step5":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.209953239169403   +1.751452157445828
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.209953239169403
i= 1:   +0.421843414957600   +0.209957081372561
i= 2:   +1.000000000000000   -0.209953239432118

Norm of error function:       0.209957081372561
Relative deviation:           0.000018299945556

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.053089673588449
i= 1:   +0.842138289682777
```

## 1.5 Auswertung der Berechnungen von §7.1 in [1]

Auswertung[1] der Berechnungen von §7.1A und §7.1B in [1], zum Vergleich mit den Ergebnissen von BRAESS [6], s. Abschnitt 1.6.

### 1.5.1 Auswertung §7.1A, 5. Iterationsschritt

```
Input from job file "../Jobs/para71/para71A-step5":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                a[i]                  t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.209953239200000
i= 1:   +0.421843415058694   +0.209957081391170
i= 2:   +1.000000000000000   -0.209953239069016

Norm of error function:       0.209957081391170
Relative deviation:           0.000018300512316

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.053089673604370
i= 1:   +0.842138289881197
```

[1] mit EXPAPP_EVAL

### 1.5.2 Auswertung §7.1B, 3. Iterationsschritt

```
Input from job file "../Jobs/para71/para71B-step3":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_1
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0100
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.208782418100000   +1.752424258000000
- output: terse
-----------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.208782418100000
i= 1:   +0.423419725194666   +0.212231724460045
i= 2:   +1.000000000000000   -0.204376043943717

Norm of error function:       0.212231724460045
Relative deviation:           0.037014638298371

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.052372888184664
i= 1:   +0.846252905784351
```

## 1.6 Zusammenfassung, Vergleich

Tabelle 1-1 fasst die Ergebnisse der Abschnitte 1.1 bis 1.5 zusammen:

| $\delta_{\mathbf{R}}$ | $\delta_{\mathbf{B}}$ | **iterations** | **norm of error function** best approximation | **relative deviation** |
|---|---|---|---|---|
| 0.01 | 0.01 | 4 | 0.209980053835259 | 0.000135 |
| 0.01 | 0.001 | 4 | 0.209980053835259 | 0.000135 |
| 0.001 | 0.001 | 4 | 0.209980053835259 | 0.000135 |
| 0.001 | 0.0001 | 5 | 0.209957081372561 | 0.000018 |
| Abschnitt 1.5.1 | | 5 | 0.209957081391170 | 0.000018 |
| Abschnitt 1.5.2 | | 3 | 0.212231724460045 | 0.037014 |

**Tabelle 1-1**

# 2 Approximationen zu $\mathbf{f(x) = \sqrt{x}}$ bzgl. $\mathbf{V_2^0}$

Zwei Berechnungen zur Funktion $f(x) = \sqrt{x}$ mit *unterschiedlichen* Werten für $\delta_B$:

- $\delta_R = 0.01$ $\delta_B = 0.1$:
  "best approximation" berechnet mit "iteration step 3" (Abschnitt 2.1)
- $\delta_R = 0.01$ $\delta_B = 0.01$:
  "best approximation" berechnet mit "iteration step 4" (Abschnitt 2.2).

Abschnitt 2.3 fasst Auswertung der Abschnitte 2.1 bis 2.2 zusammen.

**Hinweis:**
Die Berechnung von Abschnitt 2.2 ist identisch mit

- der Berechnung von Abschnitt 3.12
- der Beispielrechnung der RZ-Programmdokumentation [5], p. 7 ff.

## 2.1 Berechnung mit $\delta_R = 0.01$, $\delta_B = 0.1$

### 2.1.1 Die Berechnung

```
Input from job file "../Jobs/RZ/RZ-sqrt-terse-plots-delta_01":
----------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-01
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -5.000000000000000
------------  End Of Initialization  -----------
```

```
BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.350521013348921   +0.976003735173723 *
* i= 2:   -0.220863724854483   -5.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.410734043041571   +0.905518368658465 *
* i= 2:   -0.377277778939087  -12.759396505326684 *
*************************************************

BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.454746483503314   +0.813913434785068 *
* i= 2:   -0.423122395518619   -8.539352776514571 *
*************************************************
```

```
BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460356754157183   +0.805258269612300 *
* i= 2:   -0.430351920506976   -9.432526559173846 *
*************************************************
               END OF Braess ITERATION:
       a best approximation has been calculated
                 with delta_B = 1.000000E-01
```

### 2.1.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 0:**

```
Norm of error function:       0.156366067711090
Relative deviation:           0.170809304138800
```

**Auswertung Iteration Step 1:**

```
Norm of error function:       0.067619064275582
Relative deviation:           0.765865620000574
```

**Auswertung Iteration Step 2:**

```
Norm of error function:       0.036306314356922
Relative deviation:           0.686512178253907
```

**Auswertung Iteration Step 3:**

```
The local extrema of f-E(a) in interval I:
-----------------------------------------
               x[i]                  y[i]
         ------------------   ------------------
i= 0:   +0.000000000000000   -0.030004833650207
i= 1:   +0.017013898905620   +0.030274269482570
i= 2:   +0.178011069268428   -0.029117848783157
i= 3:   +0.634027109553937   +0.030291872924858
i= 4:   +1.000000000000000   -0.029909848071901

Norm of error function:       0.030291872924858
Relative deviation:           0.038757066775420
```

## 2.2 Berechnung mit $\delta_{\mathbf{R}} = 0.01$, $\delta_{\mathbf{B}} = 0.01$

Diese Berechnung ist in den Ergebnissen identisch mit der Beispielrechnung der RZ-Programmdokumentation [5], p. 7 ff.

### 2.2.1 Die Berechnung

```
 Input from job file "../Jobs/RZ/RZ-sqrt-terse-plots-delta_001":
 ---------------------------------------------------------------
 - Function: f(x)=sqrt(x)
 - Approximation with respect to V_2
 - Interval                      : [0.00,1.00]
 - Distance of equidistant points: 0.0100
 - Braess termination criterion  : 1.000000E-02
 - Remez  termination criterion  : 1.000000E-02
 - Plot-Indicator                : 1
 - output                        : terse
 - Starting parameters:
                 a[i]                 t[i]
   i= 1:   +0.209953239200000   +1.751452157000000
   i= 2:   +0.000000000000000   -5.000000000000000
------------  End Of Initialization  -----------

 BRAESS - iteration step 0
 Factor C=0.500000 yields better approximation:
 ************************************************
 * BRAESS-iteration terminated with approximation: *
 *                 a[i]                 t[i]      *
 *         ------------------   ------------------ *
 * i= 1:   +0.350521013348921   +0.976003735173723 *
 * i= 2:   -0.220863724854483   -5.000000000000000 *
 ************************************************

 BRAESS - iteration step 1
 Factor C=1.000000 yields better approximation:
 ************************************************
 * BRAESS-iteration terminated with approximation: *
 *                 a[i]                 t[i]      *
 *         ------------------   ------------------ *
 * i= 1:   +0.410734043041571   +0.905518368658465 *
 * i= 2:   -0.377277778939087  -12.759396505326684 *
 ************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.454746483503314   +0.813913434785068 *
* i= 2:   -0.423122395518619   -8.539352776514571 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460356754157183   +0.805258269612300 *
* i= 2:   -0.430351920506976   -9.432526559173846 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461137474201134   +0.803485102482800 *
* i= 2:   -0.431306616860017   -9.451802367671661 *
*************************************************
              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

### 2.2.2 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 0**

```
Norm of error function:       0.156366067711090
Relative deviation:           0.170809304138800
```

**Auswertung Iteration Step 1**

```
Norm of error function:       0.067619064275582
Relative deviation:           0.765865620000574
```

**Auswertung Iteration Step 2**

```
Norm of error function:       0.036306314356922
Relative deviation:           0.686512178253907
```

**Auswertung Iteration Step 3**

```
Norm of error function:       0.030291872924858
Relative deviation:           0.038757066775420
```

**Auswertung Iteration Step 4**

```
The local extrema of f-E(a) in interval I:
-----------------------------------------
               x[i]                  y[i]
         ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830857341117
i= 1:   +0.016848242443383   +0.030190684735756
i= 2:   +0.178089229925229   -0.029948455971921
i= 3:   +0.635245552358696   +0.029844185859511
i= 4:   +1.000000000000000   -0.029829376198075

Norm of error function:       0.030190684735756
Relative deviation:           0.011967550283928
```

## 2.3 Zusammenfassung, Vergleich

| $\delta_{\mathbf{R}}$ | $\delta_{\mathbf{B}}$ | **iterations** | **norm of error function** iteration step 0 | **norm of error function** best approximation | **relative deviation** |
|---|---|---|---|---|---|
| 0.01 | 0.1 | 3 | | 0.030291872924858 | 0.038757 |
| 0.01 | 0.01 | 4 | | 0.030190684735756 | 0.011967 |

**Tabelle 2-1**

# 3 Approximationen zu $\mathbf{f(x) = \sqrt{x}}$ bzgl. $\mathbf{V_2^0}$

Berechnungen von Approximationen für $f(x) = \sqrt{x}$ bzgl. $V_2^0$ auf dem Intervall `[0,1]`.

In §7.2 von [1] werden - ausgehend von der Minimallösung [2] bzgl. $V_1^0$ - Startfunktionen für die Berechnung von Minimallösungen in $V_2^0$ nach dem Algorithmus von Braess angegeben für $t_2 \in [-15, +14]$.

In diesem Kapitel werden zu diesen $t_2$-Werten die Iterationsschritte des Programms BRAESS [6] angegeben

- bis zur Berechnung der Minimallösung bzw.
- bis zum Abbruch wegen Erreichens
  - der maximalen Anzahl von BRAESS-Iterationsschritten
  - der maximalen Anzahl von Halbierungen des Faktors C.

Es werden also zu "Startfunktionen" auch Minimallösungen nach Braess berechnet, vgl. hierzu die Ergebnisse [1], §7.3.

Die Berechnungen gehen aus von der in §7.1A von [1] (im 5. Iterationsschritt) berechnete Minimallösung bzgl. $V_1^0$. Diese ist auch die mit BRAESS berechnete "best approximation" von Abschnitt 1.4.
Sofern nicht anderes angegeben, wurde mit den Endekriterien $\delta_R = 0.01$, $\delta_B = 0.01$ gerechnet[3].

Die Abschnitte "Auswertung" geben für die jeweiligen Fehlerfunktion an

- die lokalen Extrema. "`(*)`" kennzeichnet dabei das Extremum mit der Norm der Fehlerfunktion
- die Nullstellen.

Abschnitt 3.32 fasst die Ergebnisse dieser Berechnungen zusammen.

Hinweis:
Die Ergebnisse von **"iteration step 0"** entsprechen den **"Startfunktionen"** von [1], §7.2.

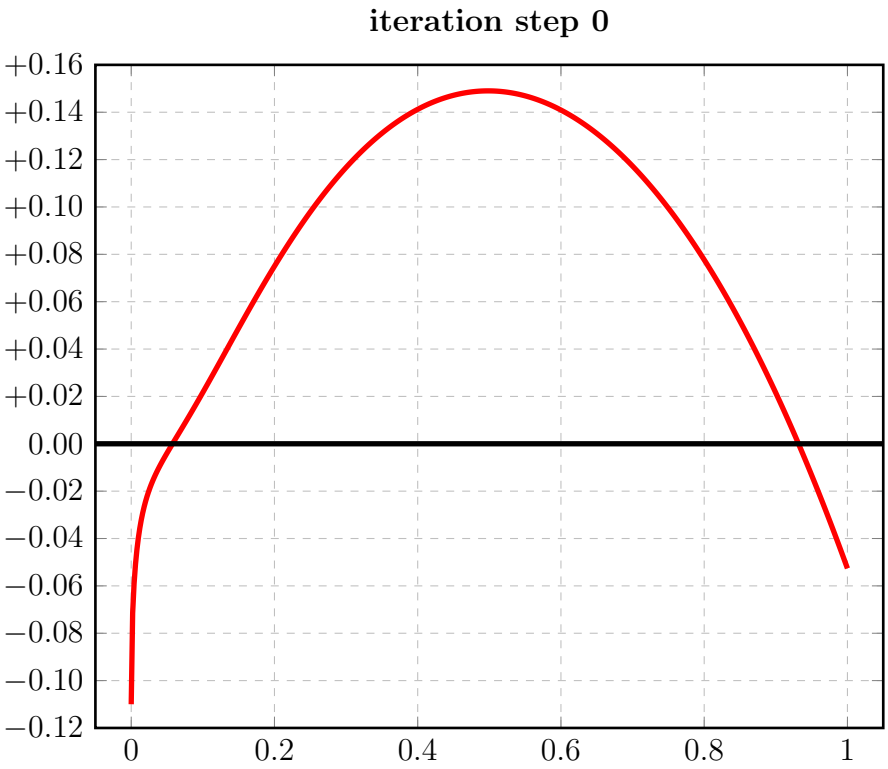
[2] s. Abschnitt 1.5.1, vgl. auch die Berechnung mit BRAESS von Abschnitt 1.4
[3] In Abschnitt 3.2 mit $t_2 = -15.0$ gilt $\delta_R = 0.001$, $\delta_B = 0.001$

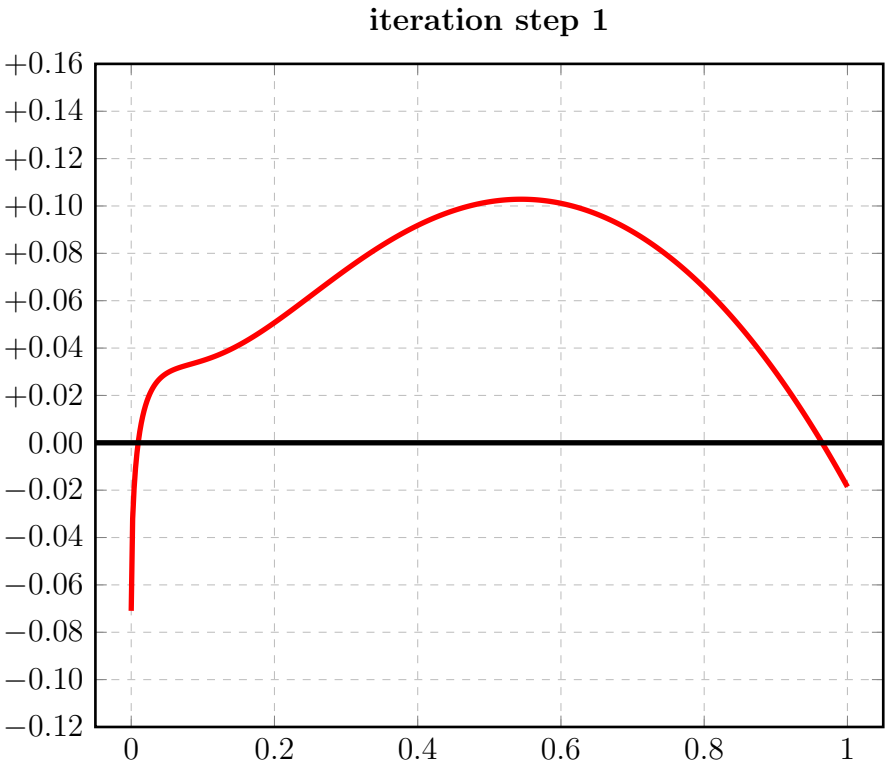
## 3.1 Berechnung 1 für $t_2 = -15.0$

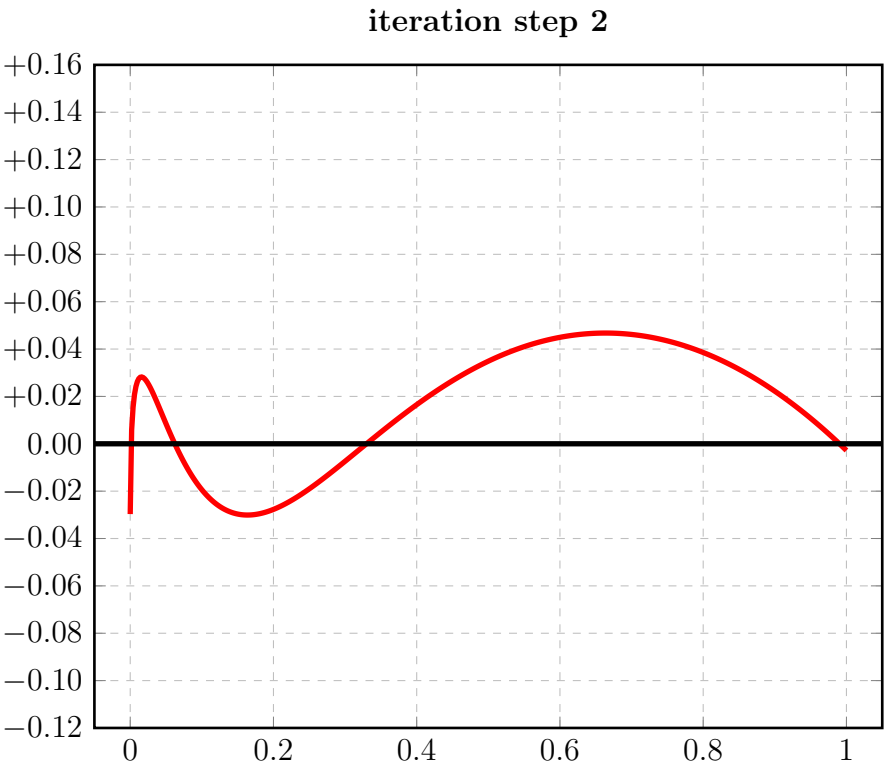
### 3.1.1 Die Berechnung

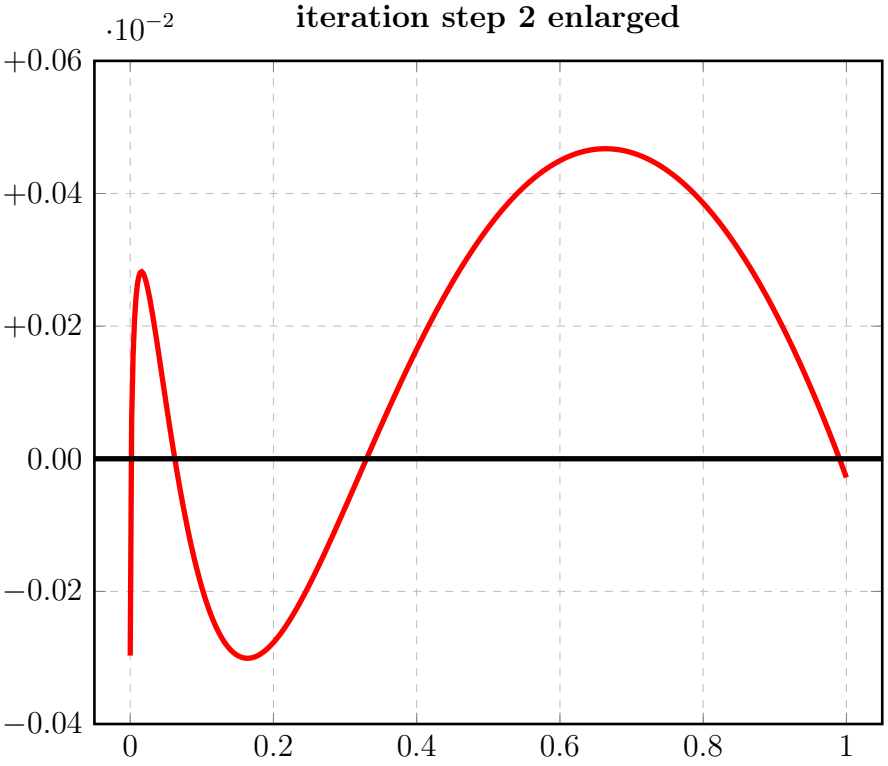
```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -15.000000000000000
------------  End Of Initialization  -----------


BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.296000781066868   +1.268686336086898 *
* i= 2:   -0.186020505217690  -15.000000000000000 *
*****************************************************

BRAESS - iteration step 1
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.364116445476594   +1.028735385125311 *
* i= 2:   -0.293159503797243   -8.761298316577019 *
*****************************************************
```

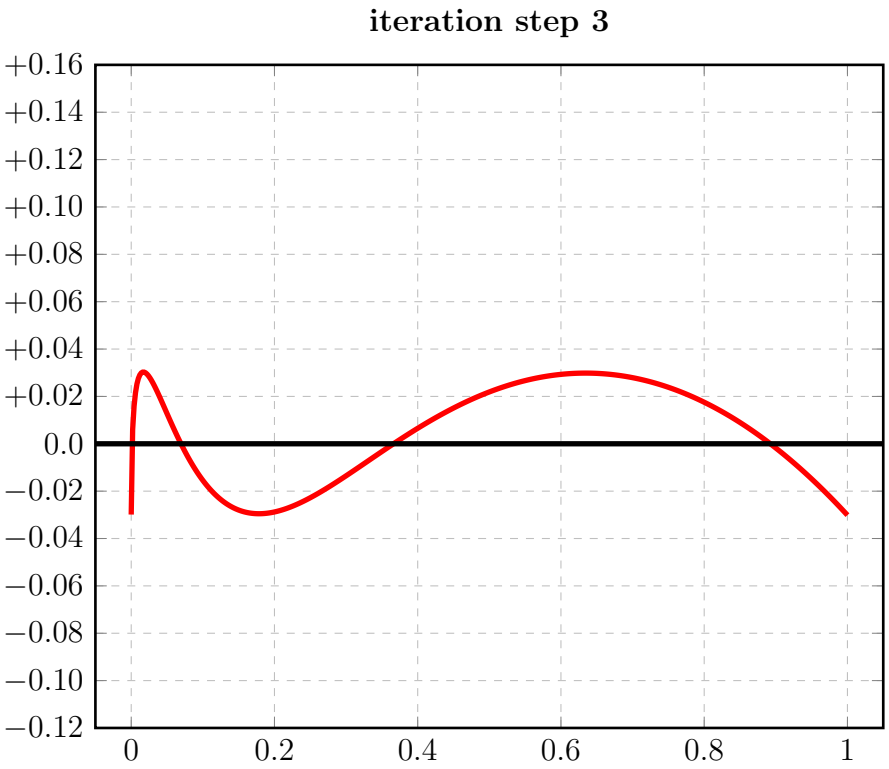
```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.454452758975408   +0.791502742589011 *
* i= 2:   -0.424770205586721  -10.000581402946368 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460913790967652   +0.804277154928400 *
* i= 2:   -0.431030448040102   -9.432055797292904 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461137802579338   +0.803485770353257 *
* i= 2:   -0.431306926815468   -9.451761297555690 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

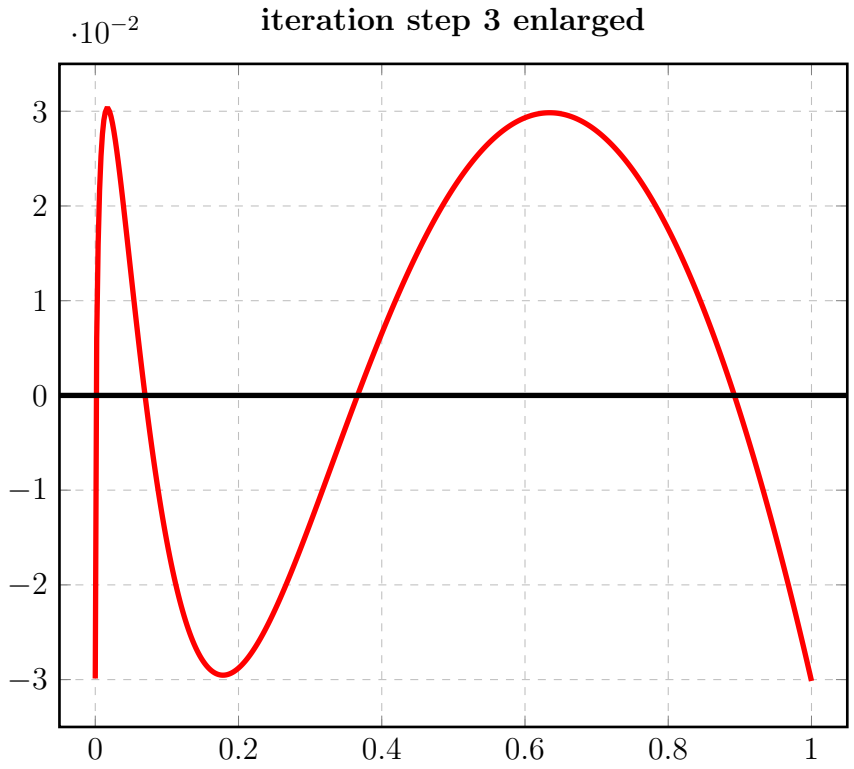

iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

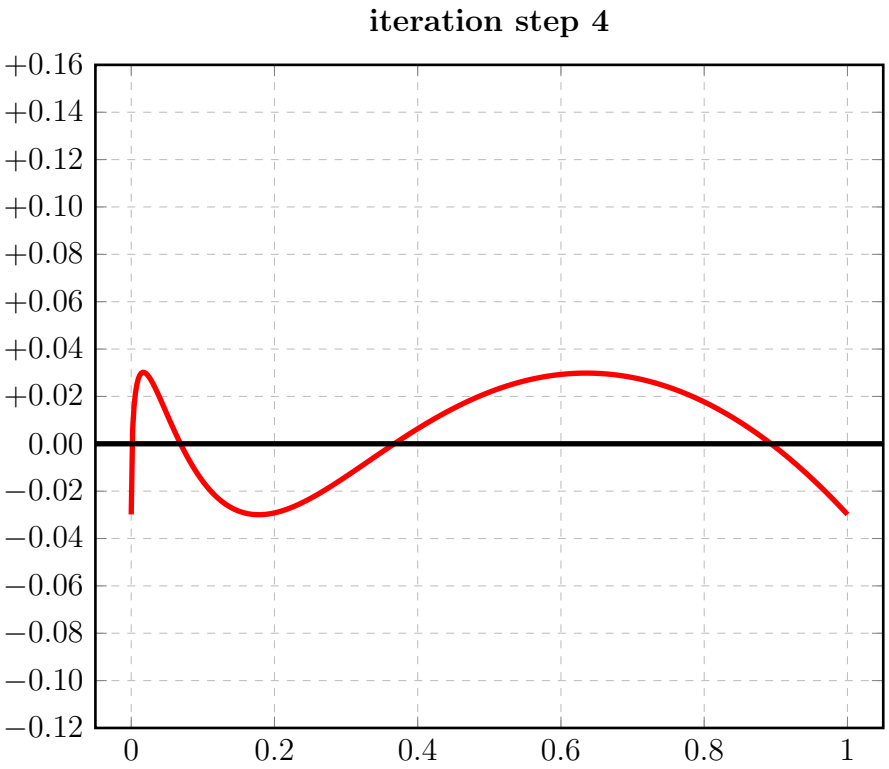

iteration step 1
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

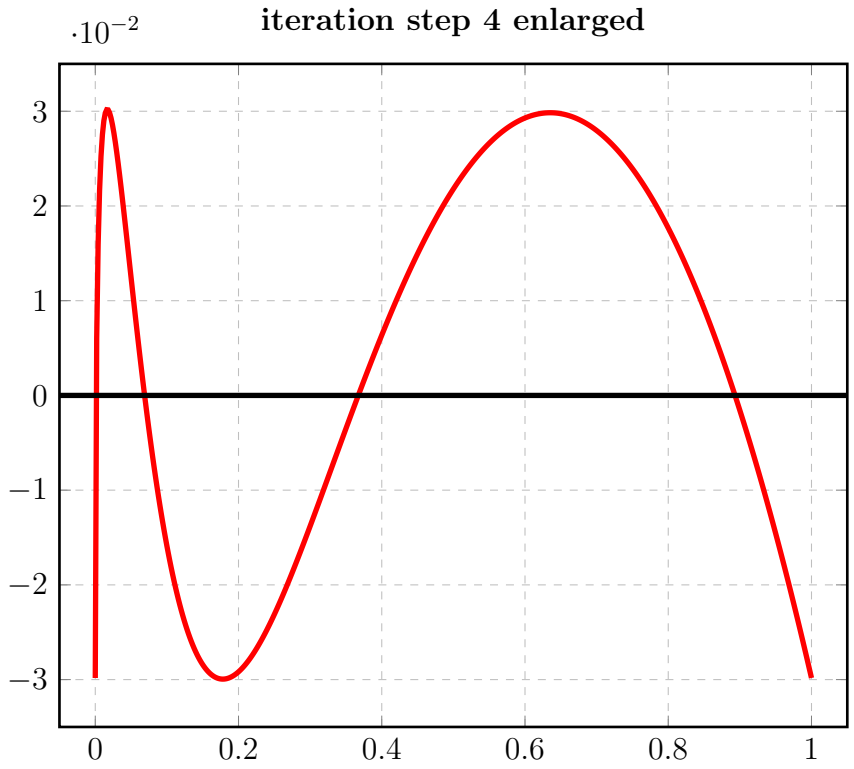

iteration step 2
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1


iteration step 2 enlarged
·10−2
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
0
0.2
0.4
0.6
0.8
1


iteration step 3
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.0
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

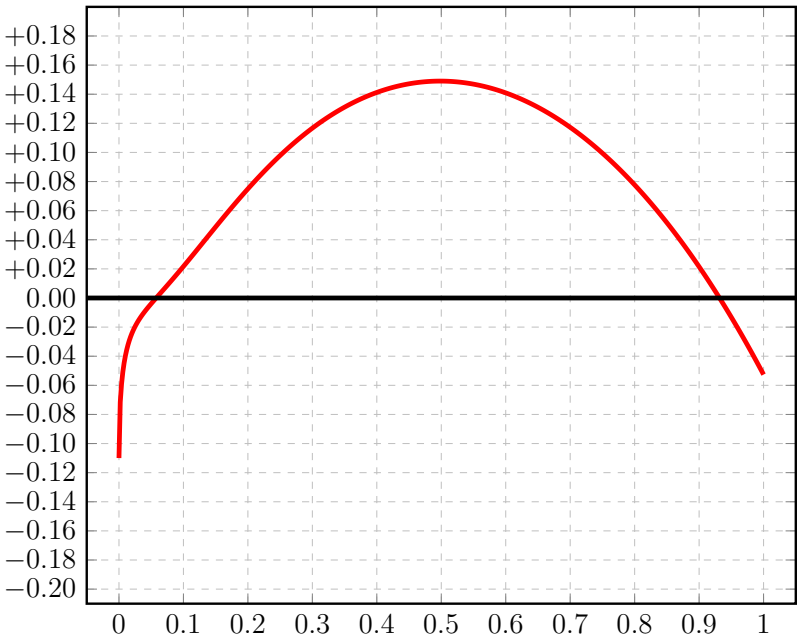

iteration step 3 enlarged
·10−2
3
2
1
0
−1
−2
−3
0
0.2
0.4
0.6
0.8
1


iteration step 4
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

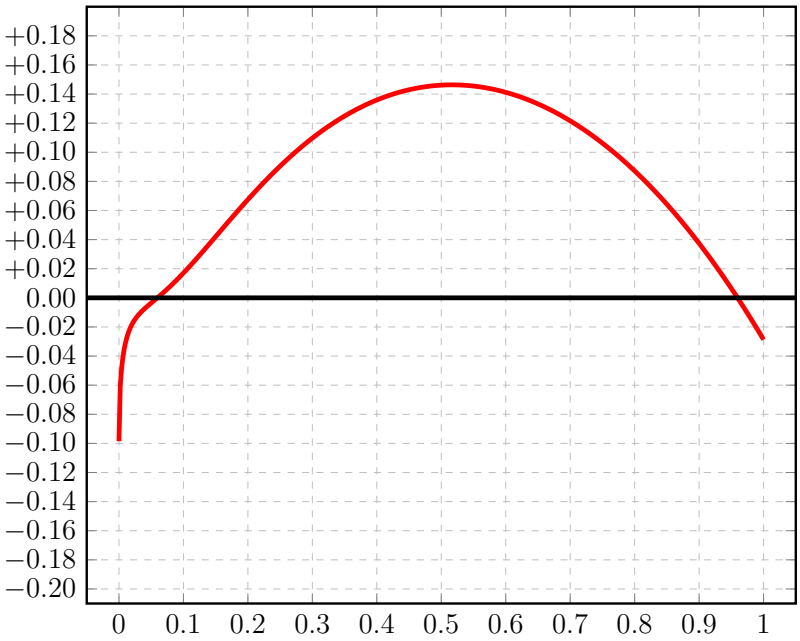

iteration step 4 enlarged
·10−2
3
2
1
0
−1
−2
−3
0
0.2
0.4
0.6
0.8
1

Zum Vergleich: die Startfunktionen von [1] nach Fall 2 (Zeichnung 14A) und Fall 1 (Zeichnung 14F):

**Zeichnung 14A von [1], §7.2**




**Zeichnung 14F von [1], §7.2**



### 3.1.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-15/t-15step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.296000781066868   +1.268686336086898
  i= 2:   -0.186020505217690  -15.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.498352003121827:
           x                  err(x)
  ------------------     ------------------
  +0.498352003121827     +0.149018547847034 (*)
  +0.000000000000000     -0.109980275849178
  +1.000000000000000     -0.052631370229810
A zero has been found at   x=  +0.057677097693992
A zero has been found at   x=  +0.931680996068709
```

### 3.1.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

```
Input from job file "../Jobs/para72/eval-t-15":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461137802579338   +0.803485770353257
  i= 2:   -0.431306926815468   -9.451761297555690
- output: terse
-------------------------------------------------

The local extrema of f-E(a) in interval I:
-----------------------------------------
               x[i]                y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830875763870
i= 1:   +0.016848360086338   +0.030190865460118
i= 2:   +0.178090234335988   -0.029948254545283
i= 3:   +0.635243942123470   +0.029843341394922
i= 4:   +1.000000000000000   -0.029830795968834

Norm of error function:       0.030190865460118
Relative deviation:           0.011926438205593


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250481958356
i= 1:   +0.068880026067049
i= 2:   +0.367244966037746
i= 3:   +0.893244360960021
```

## 3.2 Berechnung 2 für $t_2 = -15.0$

### 3.2.1 Die Berechnung mit $\delta_R = 0.001$, $\delta_B = 0.001$

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-03
- Remez  termination criterion  : 1.000000E-03
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -15.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.296013496088734   +1.268633812361774 *
* i= 2:   -0.186046707401054  -15.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.364124592563693   +1.028713139185456 *
* i= 2:   -0.293174108795899   -8.761851525876107 *
*************************************************
```

```
BRAESS - iteration step 2
  Factor C=1.000000 yields better approximation:
  ************************************************
  * BRAESS-iteration terminated with approximation: *
  *                  a[i]                 t[i]      *
  *         ------------------   ------------------ *
  * i= 1:   +0.454454673511301   +0.791502934353831 *
  * i= 2:   -0.424772253085868  -10.000280032207179 *
  ************************************************

  BRAESS - iteration step 3
  Factor C=1.000000 yields better approximation:
  ************************************************
  * BRAESS-iteration terminated with approximation: *
  *                  a[i]                 t[i]      *
  *         ------------------   ------------------ *
  * i= 1:   +0.460862142198977   +0.804414412640620 *
  * i= 2:   -0.430955159379845   -9.427167249636097 *
  ************************************************

  BRAESS - iteration step 4
  Factor C=1.000000 yields better approximation:
  ************************************************
  * BRAESS-iteration terminated with approximation: *
  *                  a[i]                 t[i]      *
  *         ------------------   ------------------ *
  * i= 1:   +0.461093824965752   +0.803601824306541 *
  * i= 2:   -0.431241786690276   -9.447352602264164 *
  ************************************************

               END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-03
```

### 3.2.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-15_X/t-15_Xstep0_x":
------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.296013496088734   +1.268633812361774
  i= 2:   -0.186046707401054  -15.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
------------------------------------------------------

A local extremum has been found at x=  +0.498362851047981:
           x                  err(x)
  ------------------     ------------------
  +0.498362851047981     +0.149009215899198 (*)
  +0.000000000000000     -0.109966788687680
  +1.000000000000000     -0.052621298054483
A zero has been found at   x=  +0.057680446745915
A zero has been found at   x=  +0.931689166110083
```

### 3.2.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

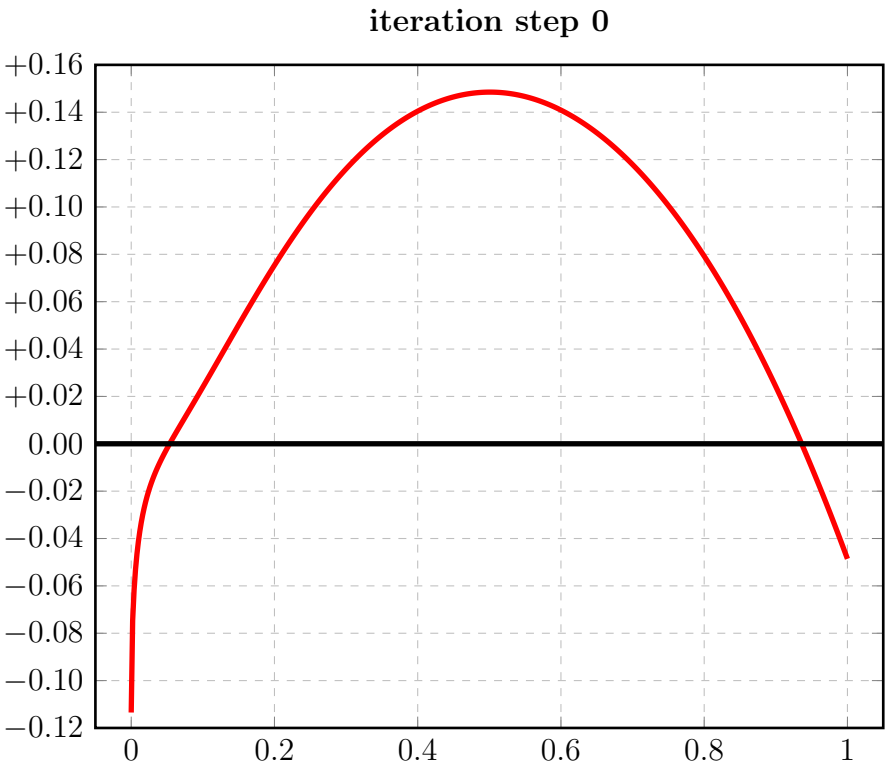
```
Input from job file "../Jobs/para72/eval-t-15_X":
--------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461093824965752   +0.803601824306541
  i= 2:   -0.431241786690276   -9.447352602264164
- output: terse
--------------------------------------------------

The local extrema of f-E(a) in interval I:
------------------------------------------
               x[i]                y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029852038275476
i= 1:   +0.016871244207082   +0.030206317120479
i= 2:   +0.178135204181001   -0.029857689165340
i= 3:   +0.635134637561161   +0.029862803302693
i= 4:   +1.000000000000000   -0.029851951076686

Norm of error function:       0.030206317120479
Relative deviation:           0.011731521005349


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001252337798396
i= 1:   +0.069005826365021
i= 2:   +0.366983825847174
i= 3:   +0.893212013634800
```

## 3.3 Berechnung für $t_2 = -14.0$

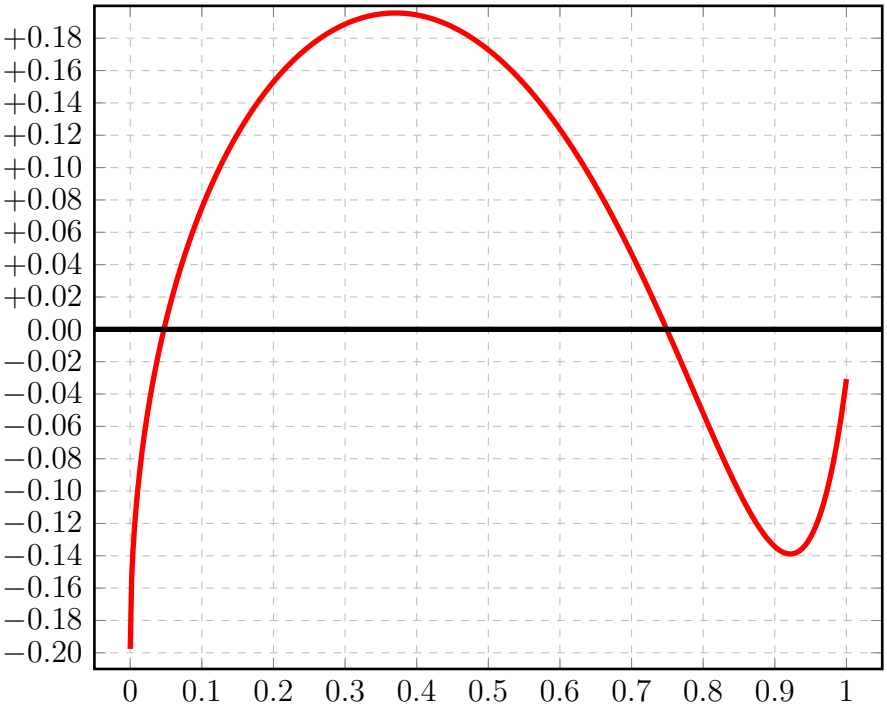
### 3.3.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -14.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.297769889565424   +1.258878524414227 *
* i= 2:   -0.184293376507797  -14.000000000000000 *
*****************************************************

BRAESS - iteration step 1
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.366365707732853   +1.020055610561084 *
* i= 2:   -0.294203156939416   -9.128730611344885 *
*****************************************************
```

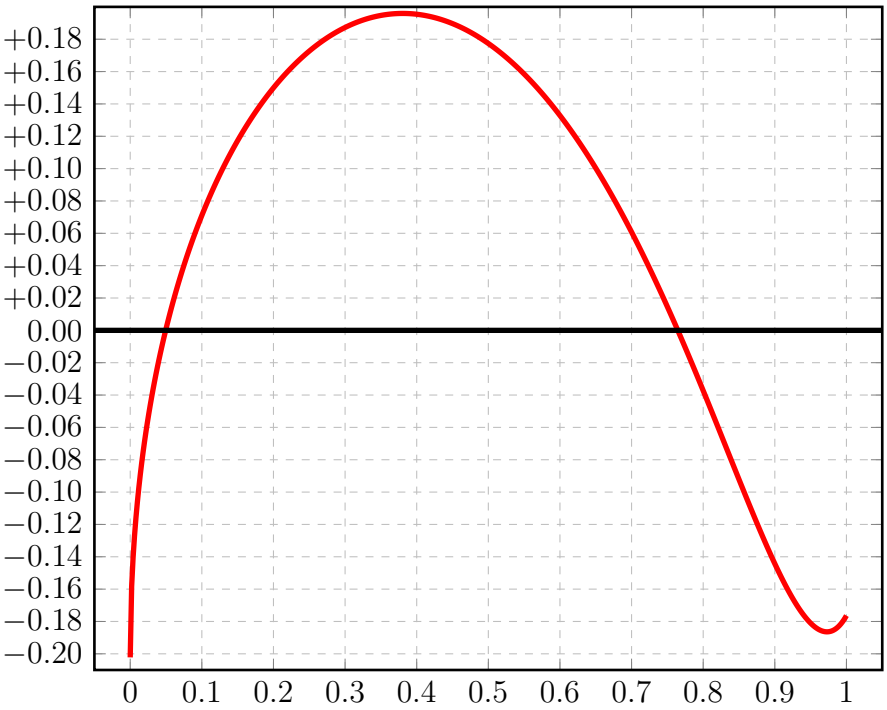
```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.455271363644993   +0.790742666667053 *
* i= 2:   -0.425664055687386   -9.843606316899212 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461009187146994   +0.804039330224560 *
* i= 2:   -0.431152132401004   -9.441246726614256 *
*                                                 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461138051461387   +0.803485258702442 *
* i= 2:   -0.431307223433962   -9.451761840813203 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

**Zeichnung 14E**




**Zeichnung 14J**



### 3.3.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-14/t-14step0_x":
-----------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.297769889565424   +1.258878524414227
  i= 2:   -0.184293376507797  -14.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.500843079348299:
           x                  err(x)
  ------------------     ------------------
  +0.500843079348299     +0.148492912675583 (*)
  +0.000000000000000     -0.113476513057627
  +1.000000000000000     -0.048587588512305
A zero has been found at   x=  +0.053636288288898
A zero has been found at   x=  +0.936075148400232
```

### 3.3.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

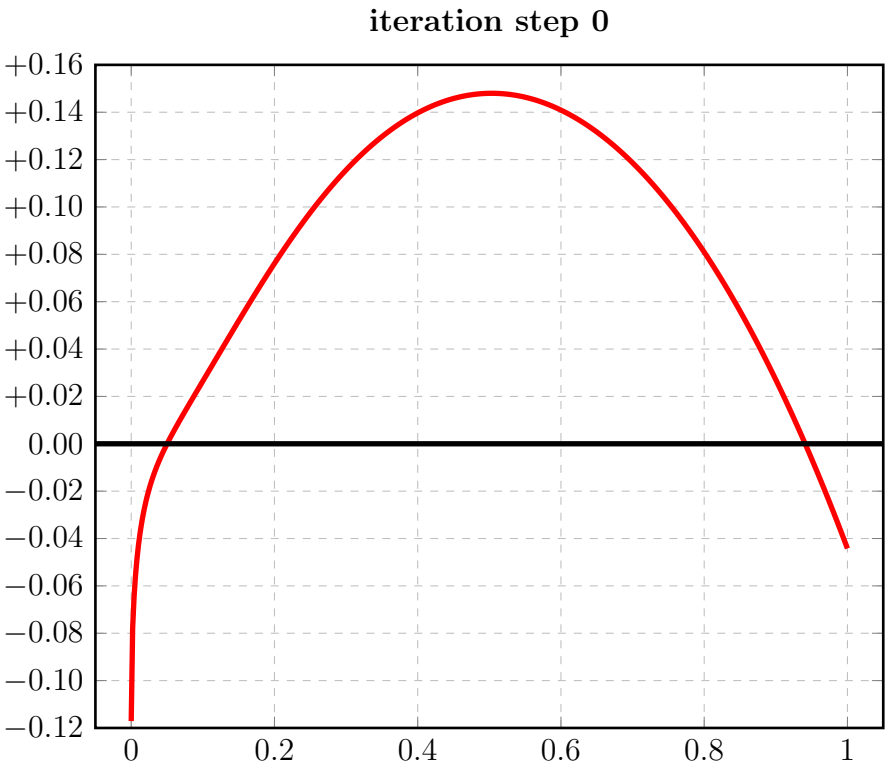
```
Input from job file "../Jobs/para72/eval-t-14":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461138051461387   +0.803485258702442
  i= 2:   -0.431307223433962   -9.451761840813203
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830828027425
i= 1:   +0.016848329274204   +0.030190866800890
i= 2:   +0.178090352062162   -0.029948445881723
i= 3:   +0.635244228860273   +0.029843176825776
i= 4:   +1.000000000000000   -0.029830824864004

Norm of error function:       0.030190866800890
Relative deviation:           0.011925525002663


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250477440114
i= 1:   +0.068879863446359
i= 2:   +0.367246019361390
i= 3:   +0.893244023160345
```

## 3.4 Berechnung für $t_2 = -13.0$

### 3.4.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -13.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.299677260983891   +1.248306857398412 *
* i= 2:   -0.182595281493066  -13.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.437502002279658   +0.774753965712508 *
* i= 2:   -0.407544321188774   -5.914698849102740 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.437681818374362   +0.858724567907573 *
* i= 2:   -0.405560110567467   -9.277331504564856 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460709522667713   +0.803069607386375 *
* i= 2:   -0.430890676757361   -9.474317353525141 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461137777822462   +0.803486391159436 *
* i= 2:   -0.431306877820264   -9.451720285197641 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
0
0.2
0.4
0.6
0.8
1

### 3.4.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-13/t-13step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.299677260983891   +1.248306857398412
  i= 2:   -0.182595281493066  -13.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.503420317492921:
           x                  err(x)
  ------------------     ------------------
  +0.503420317492921     +0.147993198537862 (*)
  +0.000000000000000     -0.117081979490825
  +1.000000000000000     -0.044206515849726
A zero has been found at   x=  +0.050080020551064
A zero has been found at   x=  +0.940982249860666
```

### 3.4.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

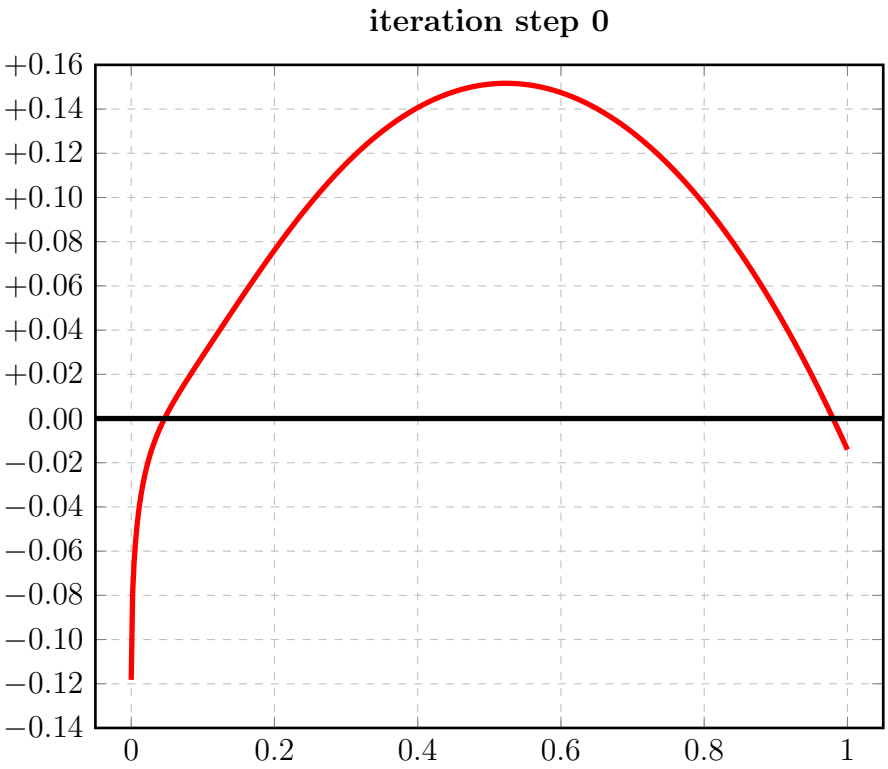
```
Input from job file "../Jobs/para72/eval-t-13":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461137777822462   +0.803486391159436
  i= 2:   -0.431306877820264   -9.451720285197641
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830900002198
i= 1:   +0.016848516691196   +0.030191098039131
i= 2:   +0.178090955078479   -0.029947708701715
i= 3:   +0.635242931003283   +0.029843107289223
i= 4:   +1.000000000000000   -0.029831378639857

Norm of error function:       0.030191098039131
Relative deviation:           0.011930604063041


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250482177697
i= 1:   +0.068881030542360
i= 2:   +0.367244505426750
i= 3:   +0.893242328968319
```

## 3.5 Berechnung für $t_2 = -12.0$

### 3.5.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -12.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.305176789173290   +1.200830606585133 *
* i= 2:   -0.186950734815751  -12.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.442410187398485   +0.766428717032093 *
* i= 2:   -0.412855585717262   -7.302039564898552 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.455331951486822   +0.819454762983039 *
* i= 2:   -0.424648463753649   -9.382498638694956 *
*                                                 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461100141149953   +0.803492578061065 *
* i= 2:   -0.431269982836206   -9.453707053193522 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
            with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.5.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-12/t-12step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.305176789173290   +1.200830606585133
  i= 2:   -0.186950734815751  -12.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.523303498983507:
           x                  err(x)
  ------------------     ------------------
  +0.523303498983507     +0.151661109503686 (*)
  +0.000000000000000     -0.118226054357539
  +1.000000000000000     -0.014063412489433
A zero has been found at   x=  +0.046595647854121
A zero has been found at   x=  +0.979927834502569
```

### 3.5.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

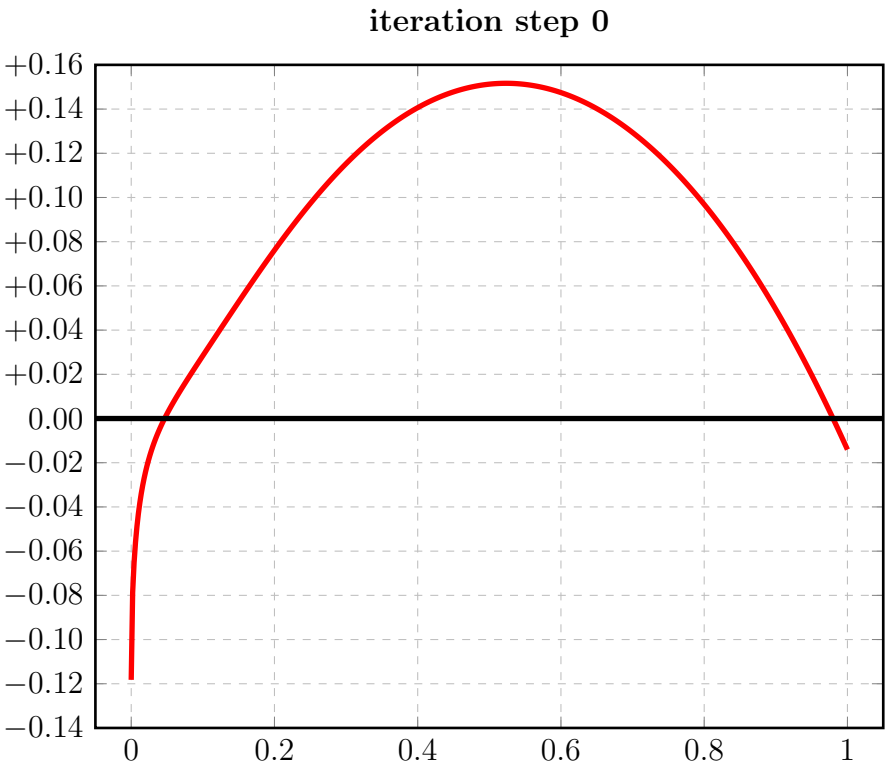
```
Input from job file "../Jobs/para72/eval-t-12":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461100141149953   +0.803492578061065
  i= 2:   -0.431269982836206   -9.453707053193522
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830158313747
i= 1:   +0.016844979633423   +0.030185425056236
i= 2:   +0.178032289585252   -0.029940070004399
i= 3:   +0.635304003534092   +0.029901357688937
i= 4:   +1.000000000000000   -0.029753765462309

Norm of error function:       0.030185425056236
Relative deviation:           0.014300265546134


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250466715762
i= 1:   +0.068863972853464
i= 2:   +0.367054162639716
i= 3:   +0.893552407969347
```

## 3.6 Berechnung für $t_2 = -11.0$

### 3.6.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -11.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.311124005237956   +1.171278293211500 *
* i= 2:   -0.192181167099855  -11.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.445602111874347   +0.764473271643888 *
* i= 2:   -0.416372356829875   -8.654708620901554 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460316411579473   +0.807604181493314 *
* i= 2:   -0.430356490399951   -9.456778142756018 *
*                                                 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461135854991347   +0.803491052349907 *
* i= 2:   -0.431305143067484   -9.451818922028327 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
            with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.6.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-11/t-11step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.311124005237956   +1.171278293211500
  i= 2:   -0.192181167099855  -11.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.531927667379111:
           x                  err(x)
  ------------------     ------------------
  +0.531927667379111     +0.149771201137898 (*)
  +0.000000000000000     -0.118942838138101
  +1.000000000000000     -0.003718275446643
A zero has been found at   x=  +0.043108165224838
A zero has been found at   x=  +0.994460035455431
```

### 3.6.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

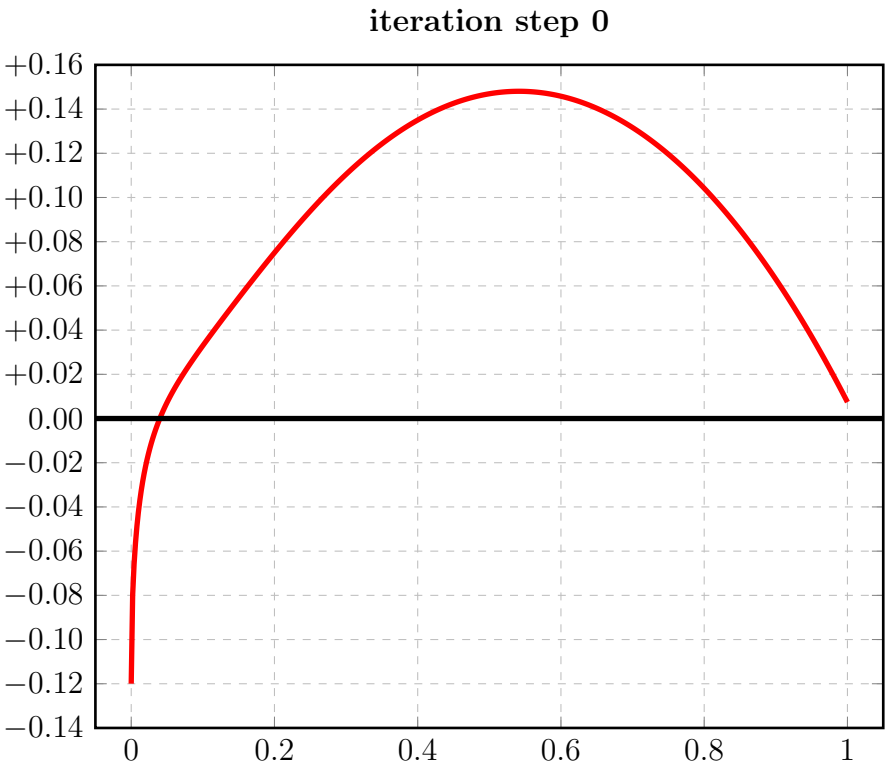
```
Input from job file "../Jobs/para72/eval-t-11":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461135854991347   +0.803491052349907
  i= 2:   -0.431305143067484   -9.451818922028327
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830711923863
i= 1:   +0.016848306779918   +0.030190919741009
i= 2:   +0.178088835310161   -0.029947661453879
i= 3:   +0.635240395019107   +0.029843964932414
i= 4:   +1.000000000000000   -0.029831888222339

Norm of error function:       0.030190919741009
Relative deviation:           0.011930998466961


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250466250360
i= 1:   +0.068880120912145
i= 2:   +0.367239607124647
i= 3:   +0.893242317249626
```

## 3.7 Berechnung für $t_2 = -10.0$

### 3.7.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                 t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  -10.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0

Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.317649223608468   +1.139367999174935 *
* i= 2:   -0.197695263502231  -10.000000000000000 *
*****************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.448320667187313   +0.764881000826390 *
* i= 2:   -0.419144027489089   -9.750020002532672 *
*****************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460870241105376   +0.805967565049982 *
* i= 2:   -0.431033412708233   -9.446278834527833 *
*                                                 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461137315673760   +0.803488446445878 *
* i= 2:   -0.431306537404263   -9.451788716913661 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
            with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.7.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-10/t-10step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.317649223608468   +1.139367999174935
  i= 2:   -0.197695263502231  -10.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.541445939632310:
           x                  err(x)
  ------------------     ------------------
  +0.541445939632310     +0.148047181239196 (*)
  +0.000000000000000     -0.119953960106237
  +1.000000000000000     +0.007420946473402
A zero has been found at   x=  +0.039927018389923
```

### 3.7.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

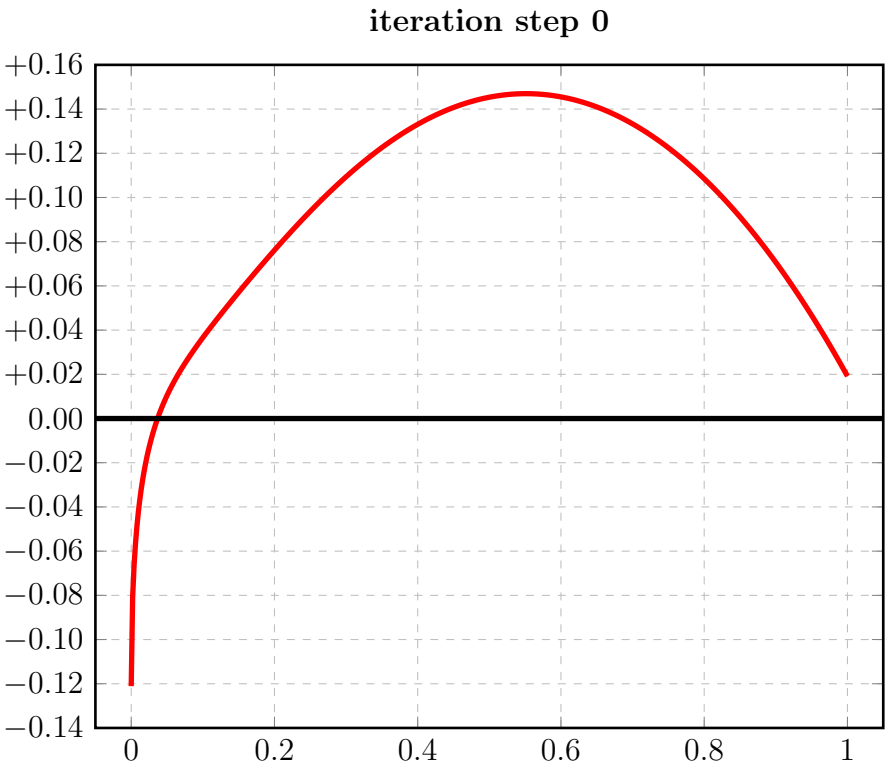
```
Input from job file "../Jobs/para72/eval-t-10":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461137315673760   +0.803488446445878
  i= 2:   -0.431306537404263   -9.451788716913661
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830778269497
i= 1:   +0.016848284111125   +0.030190835924154
i= 2:   +0.178089945168014   -0.029948409901049
i= 3:   +0.635241373145190   +0.029842827073001
i= 4:   +1.000000000000000   -0.029832465530241

Norm of error function:       0.030190835924154
Relative deviation:           0.011926057813106

The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250473087564
i= 1:   +0.068879694424019
i= 2:   +0.367245473433392
i= 3:   +0.893238897344372
```

## 3.8 Berechnung für $t_2 = -9.0$

### 3.8.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -9.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.324298163772812   +1.106776311728926 *
* i= 2:   -0.203211294628136   -9.000000000000000 *
*****************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.450090347117443   +0.768383411472037 *
* i= 2:   -0.420681368705990  -10.612648891851121 *
*****************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460050756376792   +0.807654475007759 *
* i= 2:   -0.429963713277403   -9.369012601439985 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461130706002906   +0.803501032040140 *
* i= 2:   -0.431298773783064   -9.451825436715641 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.8.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-09/t-09step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.324298163772812   +1.106776311728926
  i= 2:   -0.203211294628136   -9.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.551229074586046:
           x                  err(x)
  ------------------     ------------------
  +0.551229074586046     +0.146971002524375 (*)
  +0.000000000000000     -0.121086869144676
  +1.000000000000000     +0.019155343144224
A zero has been found at   x=  +0.036830275636690
```

### 3.8.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

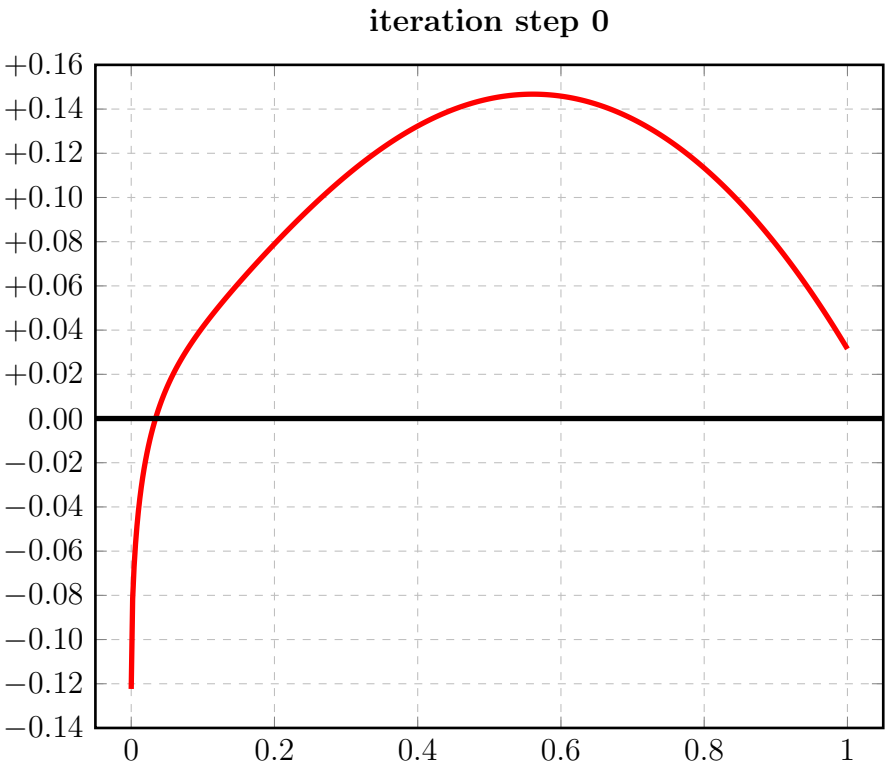
```
Input from job file "../Jobs/para72/eval-t-09":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461130706002906   +0.803501032040140
  i= 2:   -0.431298773783064   -9.451825436715641
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029831932219842
i= 1:   +0.016848903556956   +0.030190588325803
i= 2:   +0.178085898250693   -0.029943942166234
i= 3:   +0.635235360415873   +0.029847652674316
i= 4:   +1.000000000000000   -0.029830667261781

Norm of error function:       0.030190588325803
Relative deviation:           0.011921631342115


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250584275169
i= 1:   +0.068882871453457
i= 2:   +0.367217600966175
i= 3:   +0.893251509593443
```

## 3.9 Berechnung für $t_2 = -8.0$

### 3.9.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -8.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.331074279507367   +1.073499270194093 *
* i= 2:   -0.208729323620533   -8.000000000000000 *
*****************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.450190644149983   +0.776973927600496 *
* i= 2:   -0.420237306901712  -11.265337534766800 *
*****************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.459035750763184   +0.809760945901970 *
* i= 2:   -0.428601747193431   -9.268601262614521 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461107397774719   +0.803548041944127 *
* i= 2:   -0.431270809314219   -9.451776667125767 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.9.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-08/t-08step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.331074279507367   +1.073499270194093
  i= 2:   -0.208729323620533   -8.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.560772102019634:
           x                  err(x)
  ------------------     ------------------
  +0.560772102019634     +0.146740827271900 (*)
  +0.000000000000000     -0.122344955886834
  +1.000000000000000     +0.031479416442184
A zero has been found at   x=  +0.033971220954151
```

### 3.9.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

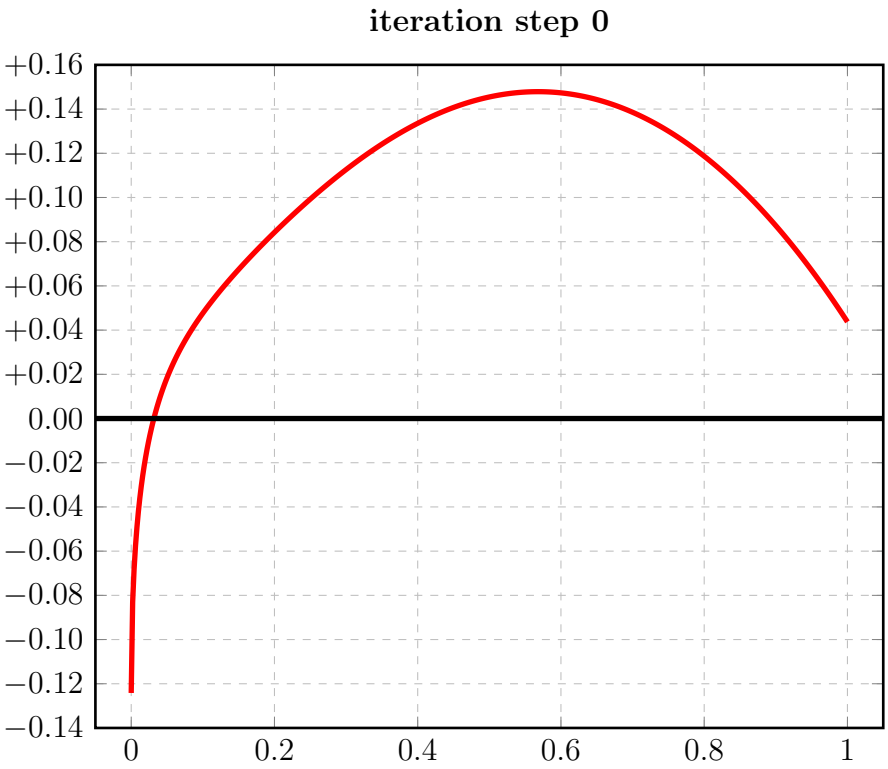
```
Input from job file "../Jobs/para72/eval-t-08":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461107397774719   +0.803548041944127
  i= 2:   -0.431270809314219   -9.451776667125767
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029836588460500
i= 1:   +0.016851805366208   +0.030190299038958
i= 2:   +0.178074586333858   -0.029926001849556
i= 3:   +0.635209705962767   +0.029863507467601
i= 4:   +1.000000000000000   -0.029827024985758

Norm of error function:       0.030190299038958
Relative deviation:           0.012032807383962


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001251026378796
i= 1:   +0.068897946102323
i= 2:   +0.367118012713320
i= 3:   +0.893286451088516
```

## 3.10 Berechnung für $t_2 = -7.0$

### 3.10.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -7.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.337393945151719   +1.041993886082899 *
* i= 2:   -0.213224547004625   -7.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.446854461701524   +0.794698765834486 *
* i= 2:   -0.416059674500454  -11.793361344944618 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.458026003565733   +0.811746303993107 *
* i= 2:   -0.427207494891373   -9.135290441218139 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461051279811871   +0.803667959029167 *
* i= 2:   -0.431202911048915   -9.451193773745613 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.10.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-07/t-07step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                a[i]                 t[i]
  i= 1:   +0.337393945151719   +1.041993886082899
  i= 2:   -0.213224547004625   -7.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------
A local extremum has been found at x=  +0.568037271883940:
           x                  err(x)
  ------------------     ------------------
  +0.568037271883940     +0.147874590729322 (*)
  +0.000000000000000     -0.124169398147094
  +1.000000000000000     +0.043728561452438
A zero has been found at   x=  +0.031610266860930
```

### 3.10.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 3:**

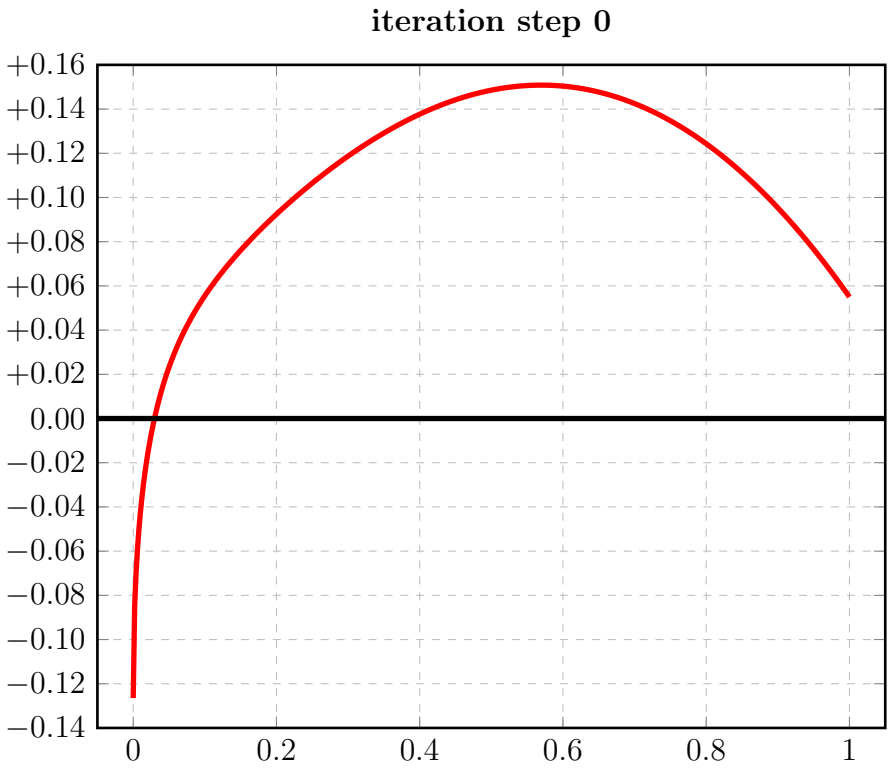
```
Input from job file "../Jobs/para72/eval-t-07":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461051279811871   +0.803667959029167
  i= 2:   -0.431202911048915   -9.451193773745613
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029848368762956
i= 1:   +0.016860574499685   +0.030191952003671
i= 2:   +0.178055531397488   -0.029876910487229
i= 3:   +0.635136474392021   +0.029898717702288
i= 4:   +1.000000000000000   -0.029825164126507

Norm of error function:       0.030191952003671
Relative deviation:           0.012148531407305


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001252123899763
i= 1:   +0.068945215186731
i= 2:   +0.366874529029245
i= 3:   +0.893346161264056
```

## 3.11 Berechnung für $t_2 = -6.0$

### 3.11.1 Die Berechnung

```
 Input from job file:
 ---------------------------------------------
 - Function: f(x)=sqrt(x)
 - Approximation with respect to V_2
 - Interval                       : [0.00,1.00]
 - Distance of equidistant points: 0.0100
 - Braess termination criterion  : 1.000000E-02
 - Remez  termination criterion  : 1.000000E-02
 - Plot-Indicator                : 1
 - output                        : terse
 - Starting parameters:
                 a[i]                t[i]
   i= 1:   +0.209953239200000   +1.751452157000000
   i= 2:   +0.000000000000000   -6.000000000000000
 ------------  End Of Initialization  -----------

 BRAESS - iteration step 0
 Factor C=0.500000 yields better approximation:
 ************************************************
 * BRAESS-iteration terminated with approximation: *
 *                  a[i]                t[i]      *
 *         ------------------   ------------------ *
 * i= 1:   +0.342793637778695   +1.014637343995412 *
 * i= 2:   -0.216300624317141   -6.000000000000000 *
 ************************************************

 BRAESS - iteration step 1
 Factor C=1.000000 yields better approximation:
 ************************************************
 * BRAESS-iteration terminated with approximation: *
 *                  a[i]                t[i]      *
 *         ------------------   ------------------ *
 * i= 1:   +0.436399877366719   +0.830452870798661 *
 * i= 2:   -0.404439620457121  -12.283679109007405 *
 ************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.456848720300864   +0.813549870299349 *
* i= 2:   -0.425606921941793   -8.921797228867163 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460852919209815   +0.804129614057162 *
* i= 2:   -0.430950356465290   -9.443758348147897 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461138071087038   +0.803485018488042 *
* i= 2:   -0.431307252029030   -9.451766926069846 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

### 3.11.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-06/t-06step0_x":
-----------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.342793637778695   +1.014637343995412
  i= 2:   -0.216300624317141   -6.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.569618928847786:
           x                  err(x)
  ------------------     ------------------
  +0.569618928847786     +0.150830093209882 (*)
  +0.000000000000000     -0.126493013461554
  +1.000000000000000     +0.054986910001453
A zero has been found at   x=  +0.029665016095782
```

### 3.11.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

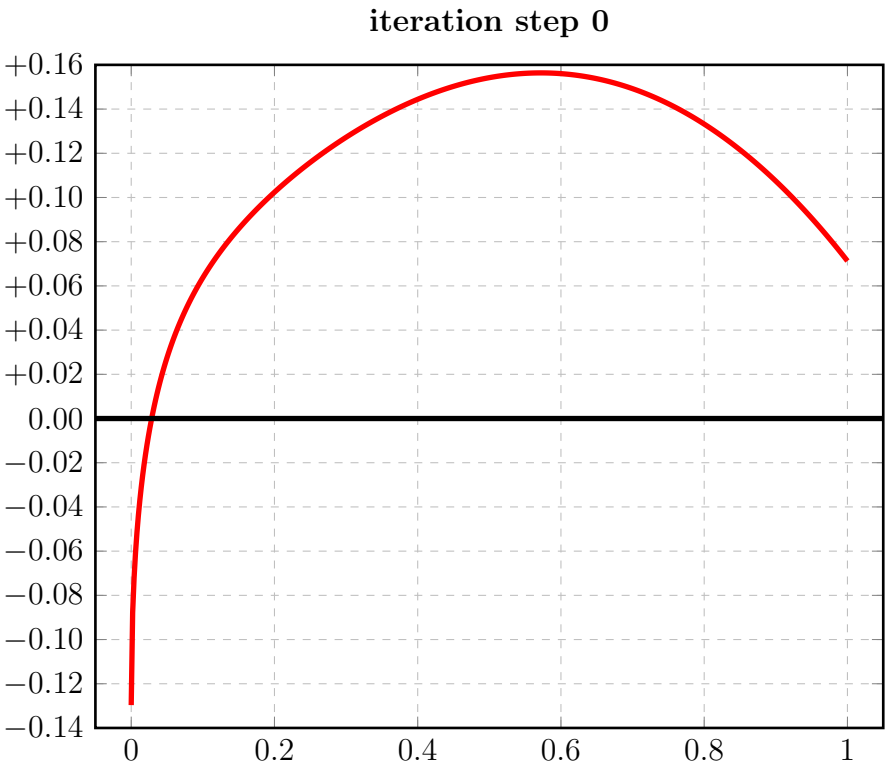
```
Input from job file "../Jobs/para72/eval-t-06":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461138071087038   +0.803485018488042
  i= 2:   -0.431307252029030   -9.451766926069846
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830819058008
i= 1:   +0.016848308417228   +0.030190841671511
i= 2:   +0.178090249967143   -0.029948513013779
i= 3:   +0.635244540597737   +0.029843257991367
i= 4:   +1.000000000000000   -0.029830621475914

Norm of error function:       0.030190841671511
Relative deviation:           0.011931439325741


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250476819784
i= 1:   +0.068879738453294
i= 2:   +0.367245998759269
i= 3:   +0.893244724273424
```

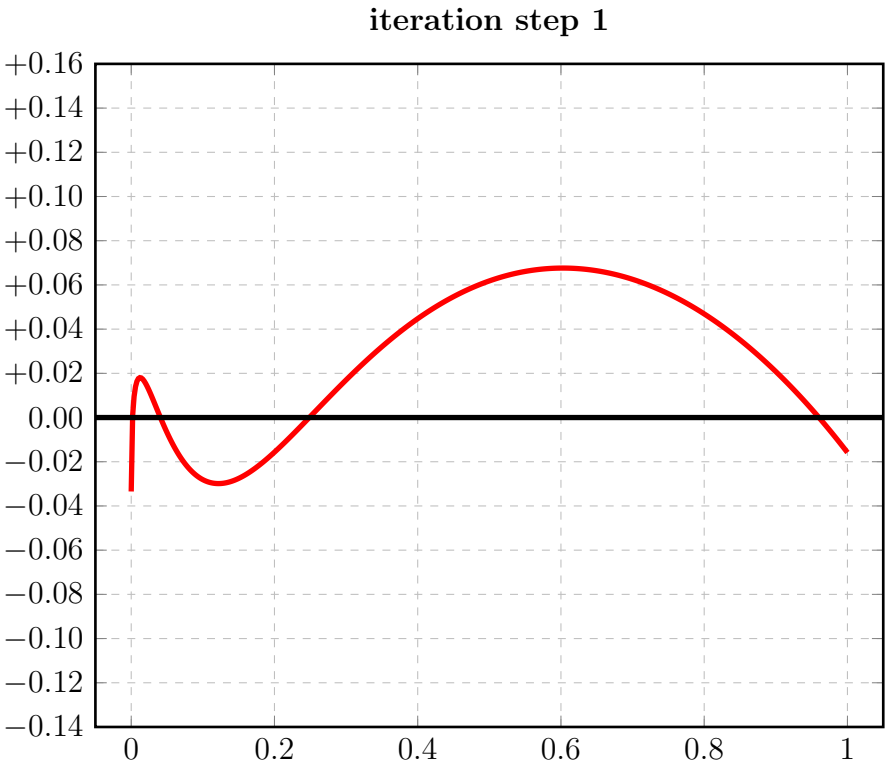
## 3.12 Berechnung für $t_2 = -5.0$

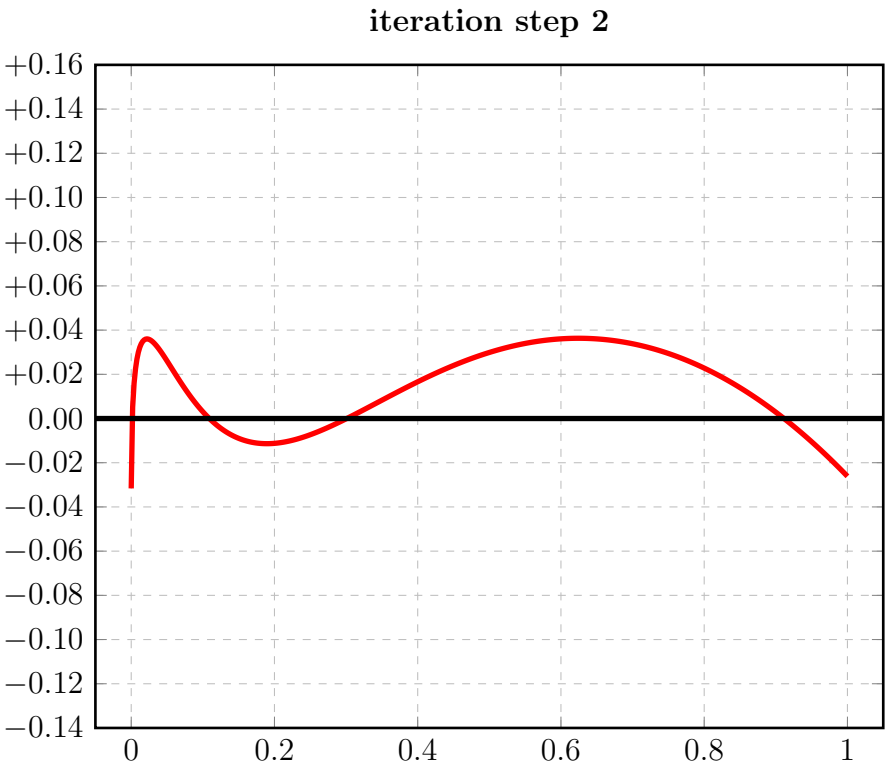
### 3.12.1 Die Berechnung

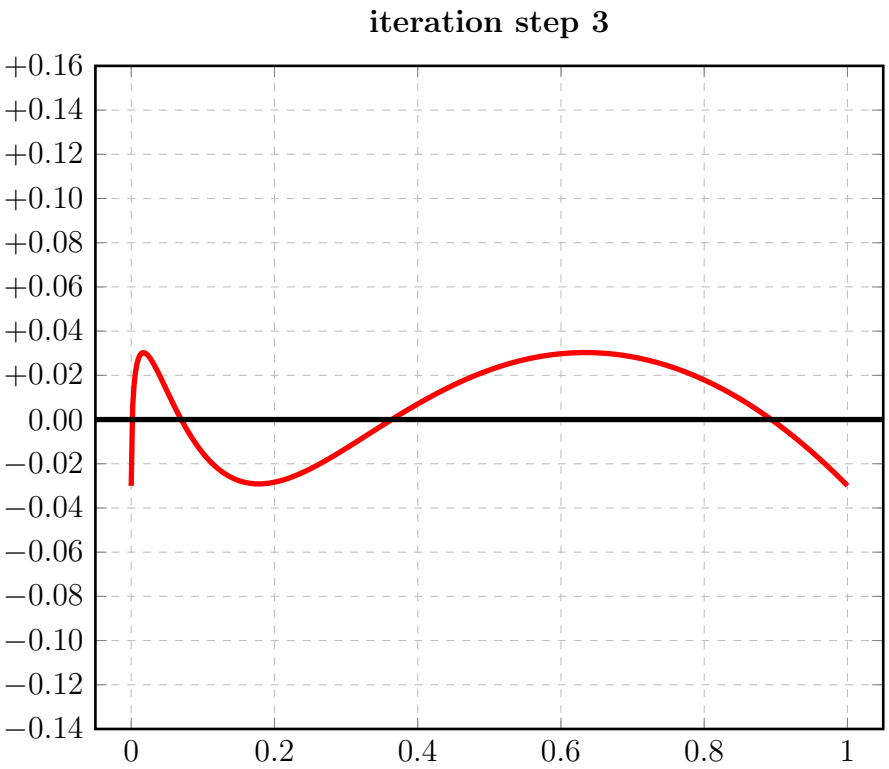
```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -5.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.350521013348921   +0.976003735173723 *
* i= 2:   -0.220863724854483   -5.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.410734043041571   +0.905518368658465 *
* i= 2:   -0.377277778939087  -12.759396505326684 *
*************************************************
```

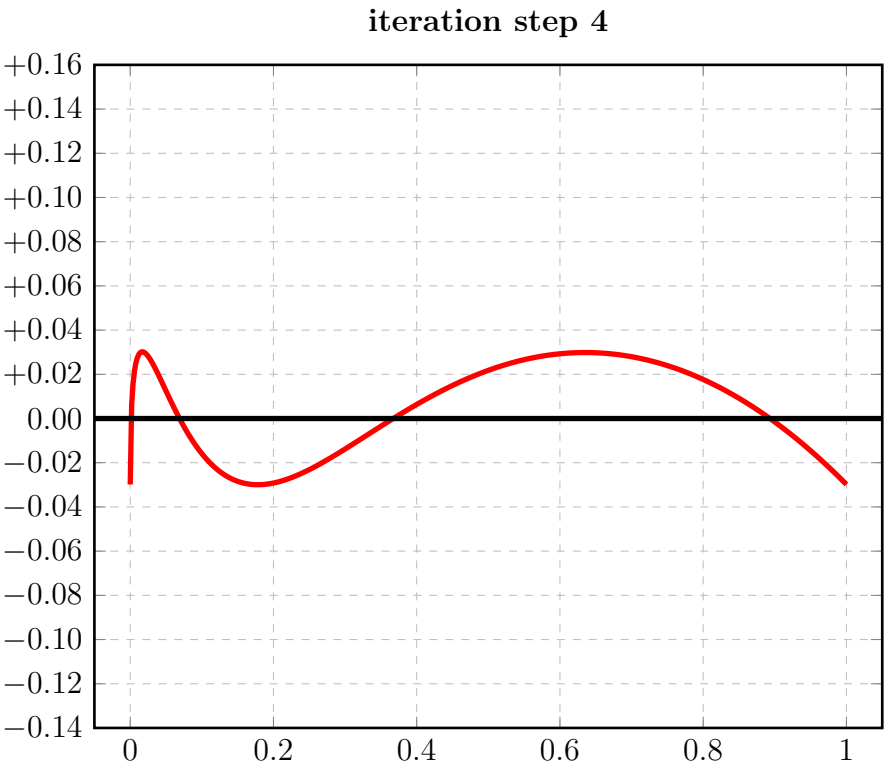
```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.454746483503314   +0.813913434785068 *
* i= 2:   -0.423122395518619   -8.539352776514571 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460356754157183   +0.805258269612300 *
* i= 2:   -0.430351920506976   -9.432526559173846 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461137474201134   +0.803485102482800 *
* i= 2:   -0.431306616860017   -9.451802367671661 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

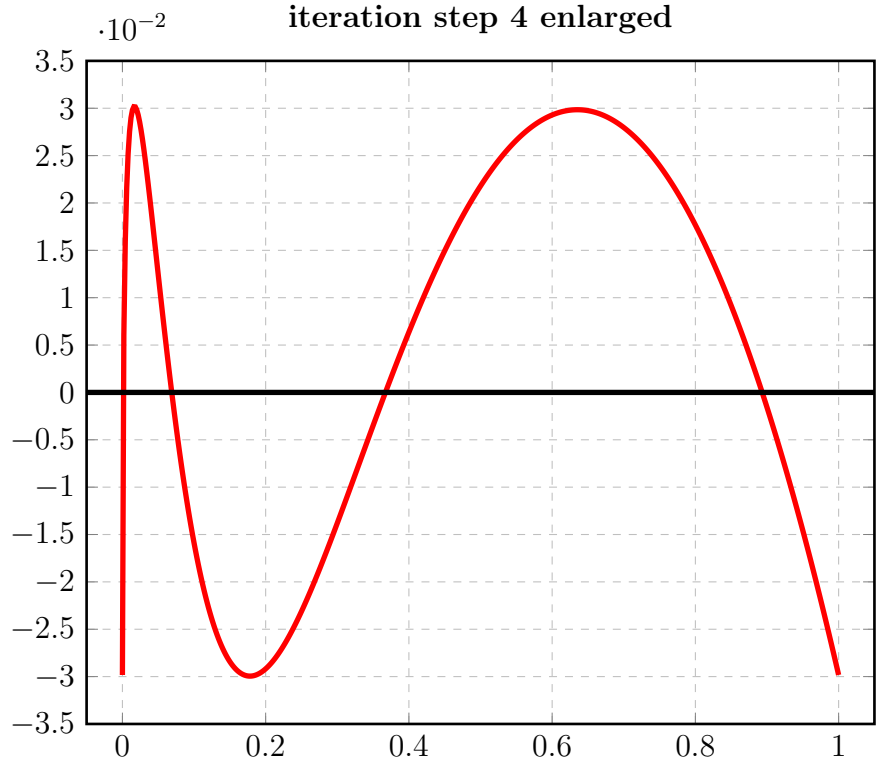

iteration step 0
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1


iteration step 2
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

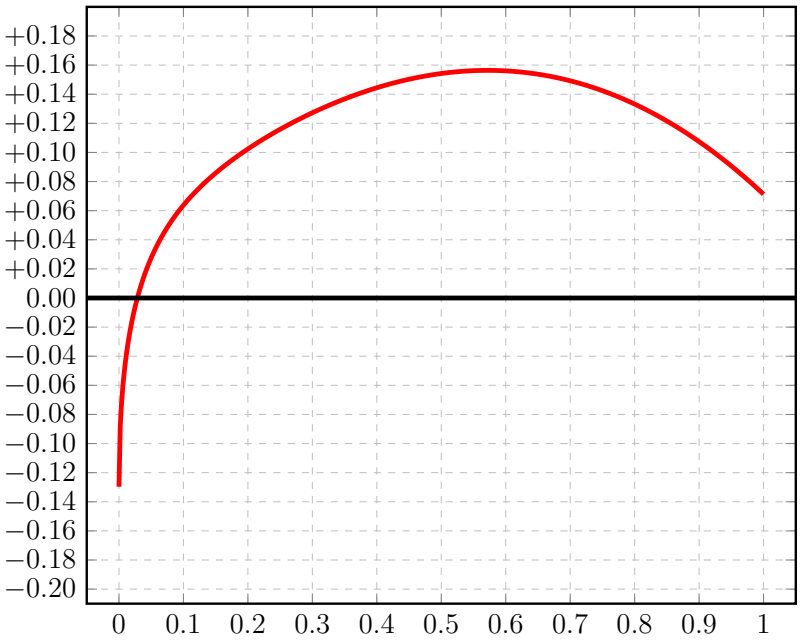

iteration step 3
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1


iteration step 4
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
0
0.2
0.4
0.6
0.8
1

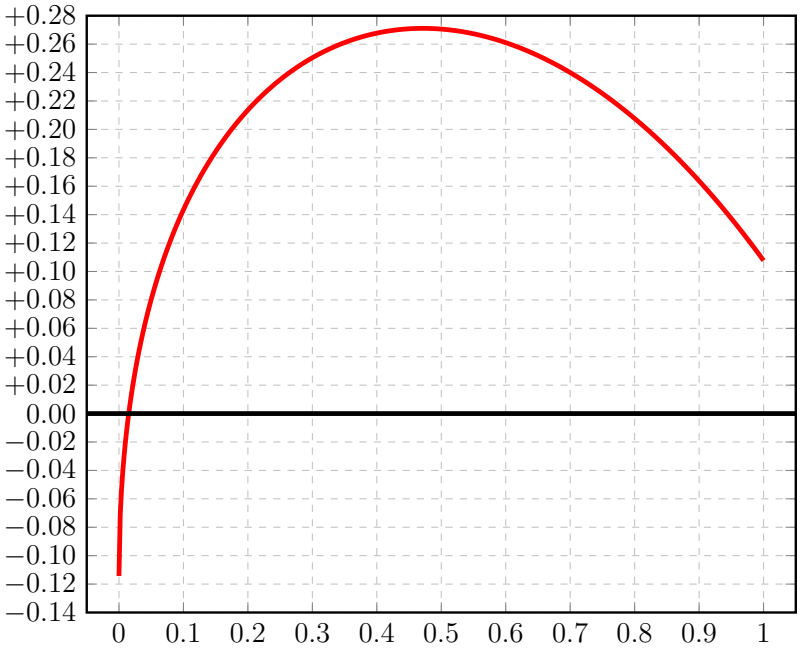

iteration step 4 enlarged
·10−2
3.5
3
2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
−3
−3.5
0
0.2
0.4
0.6
0.8
1

Zum Vergleich: die Startfunktionen von [1] nach Fall 2 (Zeichnung 14B) und Fall 1 (Zeichnung 14G):

**Zeichnung 14B von [1], §7.2**




**Zeichnung 14G von [1], §7.2**



### 3.12.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-05/t-05step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.350521013348921   +0.976003735173723
  i= 2:   -0.220863724854483   -5.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.571970319488189:
           x                  err(x)
  ------------------     ------------------
  +0.571970319488189     +0.156366067711090 (*)
  +0.000000000000000     -0.129657288494438
  +1.000000000000000     +0.071265121565663
A zero has been found at   x=  +0.028535308050202
```

### 3.12.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

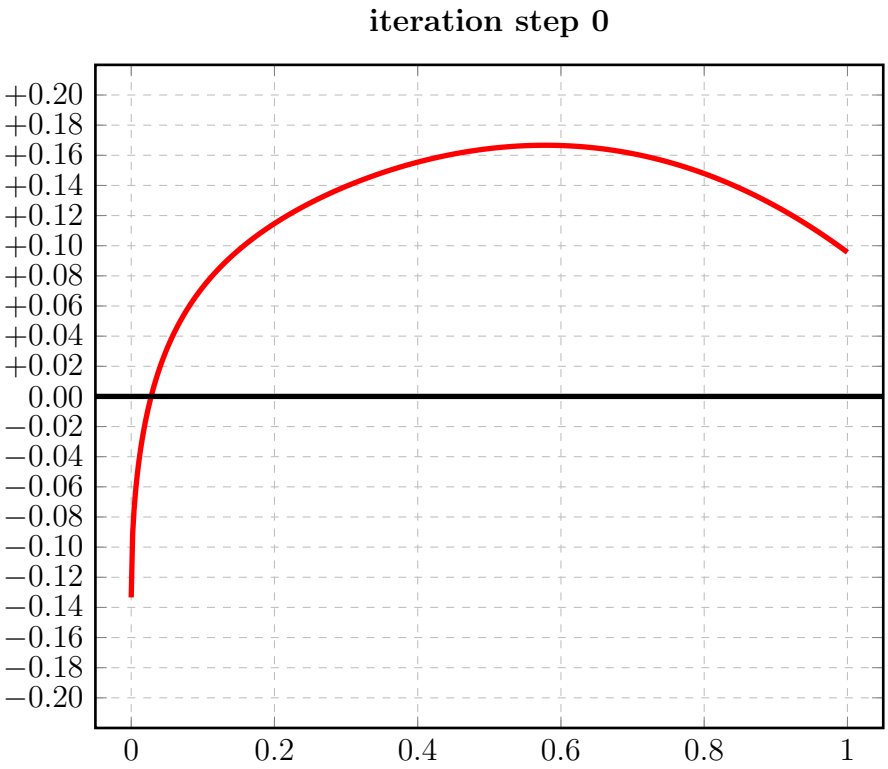
```
Input from job file "../Jobs/para72/eval-t-05":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461137474201134   +0.803485102482800
  i= 2:   -0.431306616860017   -9.451802367671661
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830857341117
i= 1:   +0.016848242443383   +0.030190684735756
i= 2:   +0.178089229925229   -0.029948455971921
i= 3:   +0.635245552358696   +0.029844185859511
i= 4:   +1.000000000000000   -0.029829376198075

Norm of error function:       0.030190684735756
Relative deviation:           0.011967550283928


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250481867009
i= 1:   +0.068879326485381
i= 2:   +0.367243066549830
i= 3:   +0.893249693715372
```

## 3.13 Berechnung für $t_2 = -4.0$

### 3.13.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -4.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.362132353948953   +0.919855295649262 *
* i= 2:   -0.228752256871569   -4.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.344826111298492   +1.070063726096432 *
* i= 2:   -0.309578816280003  -13.380011730437284 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.447593549342047   +0.798182649032354 *
* i= 2:   -0.415947918543456   -7.618883865852716 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.457452757025961   +0.812348885178880 *
* i= 2:   -0.426983643876589   -9.389082548097226 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461124604026127   +0.803482870690451 *
* i= 2:   -0.431293557865767   -9.452564760960810 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.13.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-04/t-04step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.362132353948953   +0.919855295649262
  i= 2:   -0.228752256871569   -4.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.578197600384453:
           x                  err(x)
  ------------------     ------------------
  +0.578197600384453     +0.166653412199856 (*)
  +0.000000000000000     -0.133380097077384
  +1.000000000000000     +0.095625990723534
A zero has been found at   x=  +0.027854750943059
```

### 3.13.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 4:**

```
Input from job file "../Jobs/para72/eval-t-04":

------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461124604026127   +0.803482870690451
  i= 2:   -0.431293557865767   -9.452564760960810
- output: terse
------------------------------------------------

The exact local extrema of f-E(a) in interval I:
------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029831046160360
i= 1:   +0.016846787587632   +0.030187887081252
i= 2:   +0.178066882243718   -0.029946698693369
i= 3:   +0.635272582827279   +0.029866169018083
i= 4:   +1.000000000000000   -0.029798361567982

Norm of error function:       0.030187887081252
Relative deviation:           0.012903371217149


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250525760479
i= 1:   +0.068871254183663
i= 2:   +0.367175132299438
i= 3:   +0.893371588085967
```

## 3.14 Berechnung für $t_2 = -3.0$

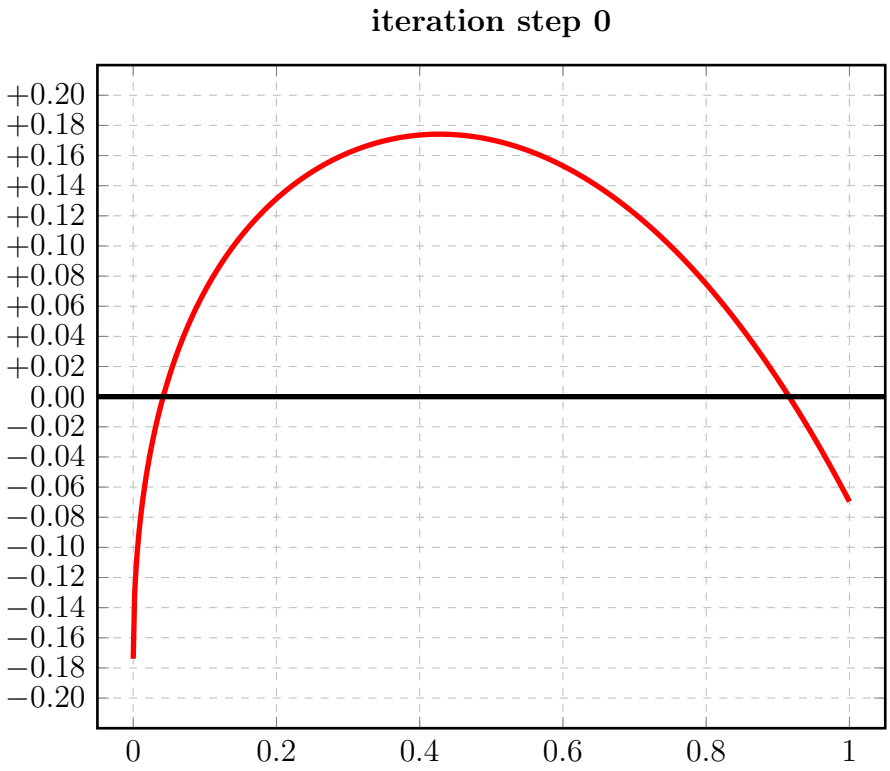
### 3.14.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -3.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.250000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.295518397487446   +1.291946465631338 *
* i= 2:   -0.121615538032537   -3.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=0.125000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.285475692749946   +1.307291742721271 *
* i= 2:   -0.128567299386339   -5.714913259481822 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.409625479124378   +0.846657383151443 *
* i= 2:   -0.377616488947097  -18.125930114126589 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.437579140090958   +0.866521217477864 *
* i= 2:   -0.398241391464169   -5.570527212042766 *
*************************************************

BRAESS - iteration step 4
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.430640919806433   +0.870465977149291 *
* i= 2:   -0.398038192745722   -9.316146045405722 *
*************************************************

BRAESS - iteration step 5
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460532012369873   +0.802475521048283 *
* i= 2:   -0.430723702730820   -9.481445898853414 *
*************************************************
```

```
BRAESS - iteration step 6
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                   a[i]                  t[i]      *
*          ------------------   ------------------ *
* i= 1:   +0.461137389937936   +0.803488654077223 *
* i= 2:   -0.431306428556656   -9.451690962314821 *
*************************************************
```

END OF Braess ITERATION:
a best approximation has been calculated
with delta_B = 1.000000E-02



### 3.14.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-03/t-03step0_x":
-----------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.295518397487446   +1.291946465631338
  i= 2:   -0.121615538032537   -3.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.427677825864559:
           x                  err(x)
  ------------------     ------------------
  +0.427677825864559     +0.174172850600343 (*)
  +0.000000000000000     -0.173902859454909
  +1.000000000000000     -0.069592053707824
A zero has been found at   x=  +0.041906600580981
A zero has been found at   x=  +0.915791347217437
```

### 3.14.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 6:**

```
Input from job file "../Jobs/para72/eval-t-03":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461137389937936   +0.803488654077223
  i= 2:   -0.431306428556656   -9.451690962314821
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830961381280
i= 1:   +0.016848662647227   +0.030191271980893
i= 2:   +0.178091480682037   -0.029947140624096
i= 3:   +0.635240413402804   +0.029842667869342
i= 4:   +1.000000000000000   -0.029832841914959

Norm of error function:       0.030191271980893
Relative deviation:           0.011934263645505


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250486426999
i= 1:   +0.068881935136566
i= 2:   +0.367243824388203
i= 3:   +0.893237516335352
```

## 3.15 Berechnung für $t_2 = -2.0$

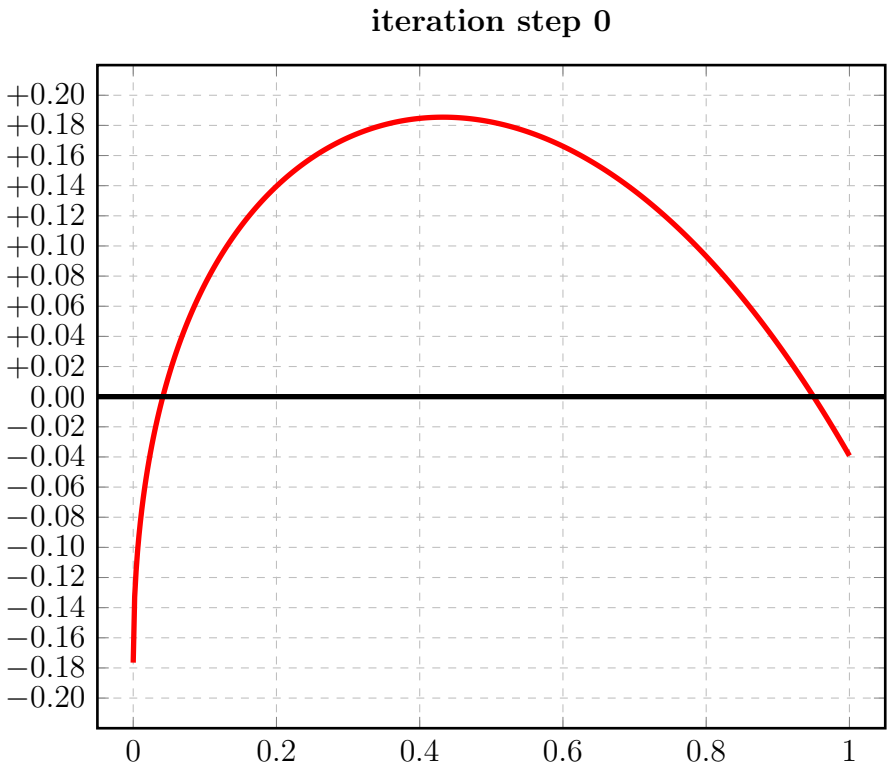
### 3.15.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -2.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.250000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.312491795114807   +1.219175843799505 *
* i= 2:   -0.136005523269045   -2.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=0.031250 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.297369750399461   +1.252241322886750 *
* i= 2:   -0.125104840226110   -2.804232110525092 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=0.062500 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.289364950592866   +1.269451566933046 *
* i= 2:   -0.125454626314049   -4.188974230956731 *
*************************************************

BRAESS - iteration step 3
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.355324379528676   +1.020379440742727 *
* i= 2:   -0.320500088664533  -20.812176041558512 *
*************************************************

BRAESS - iteration step 4
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.391248346055264   +0.948446617152976 *
* i= 2:   -0.352203734845956  -10.926350964324735 *
*************************************************

BRAESS - iteration step 5
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.457396141858236   +0.799188148353199 *
* i= 2:   -0.427320983098819   -9.128164323925814 *
*************************************************
```

```
BRAESS - iteration step 6
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                   a[i]                  t[i]      *
*          ------------------   ------------------ *
* i= 1:   +0.461053796291473   +0.803730238249606 *
* i= 2:   -0.431204359633277   -9.450824096826665 *
*************************************************
```

END OF Braess ITERATION:
a best approximation has been calculated
with delta_B = 1.000000E-02



### 3.15.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-02/t-02step0_x":
-----------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.312491795114807   +1.219175843799505
  i= 2:   -0.136005523269045   -2.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.432667931475911:
           x                  err(x)
  ------------------     ------------------
  +0.432667931475911     +0.185445188878440 (*)
  +0.000000000000000     -0.176486271845762
  +1.000000000000000     -0.039190045357145
A zero has been found at   x=  +0.041400846512828
A zero has been found at   x=  +0.949958594217723
```

### 3.15.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 6:**

```
Input from job file "../Jobs/para72/eval-t-02":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461053796291473   +0.803730238249606
  i= 2:   -0.431204359633277   -9.450824096826665
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029849436658196
i= 1:   +0.016861444601827   +0.030192437466439
i= 2:   +0.178073593827002   -0.029880169673990
i= 3:   +0.635049721744438   +0.029864397992297
i= 4:   +1.000000000000000   -0.029894913756755

Norm of error function:       0.030192437466439
Relative deviation:           0.011360487493740


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001252219075096
i= 1:   +0.068948045681279
i= 2:   +0.366952549950315
i= 3:   +0.893096870337845
```

## 3.16 Berechnung für $t_2 = -1.0$

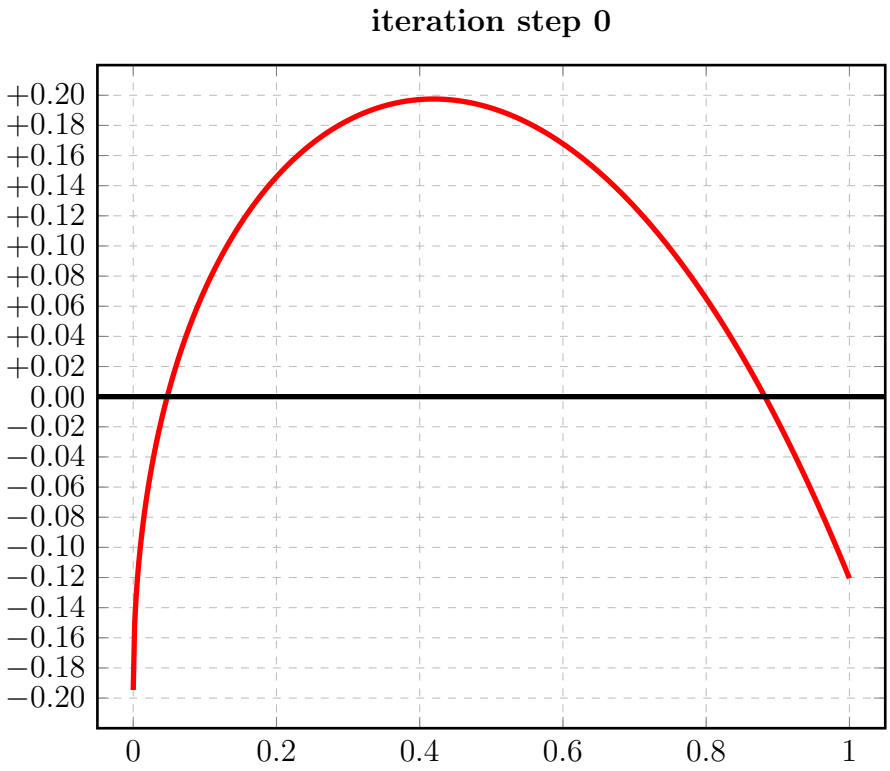
### 3.16.1 Die Berechnung

```
Input from job file:
------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -1.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.125000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.278887720646740   +1.417992614795477 *
* i= 2:   -0.084232105112638   -1.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=0.003906 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.273262822115397   +1.430277644989647 *
* i= 2:   -0.079186771929036   -1.223715676648267 *
*************************************************
```

```
BRAESS - iteration step 2
Factor C=0.003906 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.269246001756174   +1.439480085092627 *
* i= 2:   -0.075751662330164   -1.437536848891085 *
*************************************************

BRAESS - iteration step 3
Factor C=0.007812 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.263344523459912   +1.453414950346293 *
* i= 2:   -0.071018435888474   -1.846981717470040 *
*************************************************

BRAESS - iteration step 4
Factor C=0.015625 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.256727785442730   +1.469357789134876 *
* i= 2:   -0.066745972483479   -2.608553664762367 *
*************************************************

BRAESS - iteration step 5
Factor C=0.031250 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.253003326796386   +1.476040467987061 *
* i= 2:   -0.067728282869392   -3.944596296253447 *
*************************************************
```

```
BRAESS - iteration step 6
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.287921376336153   +1.288493162214647 *
* i= 2:   -0.177675629855927  -20.501304358293272 *
*************************************************

BRAESS - iteration step 7
Factor C=0.500000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.352872604133042   +1.060803749183926 *
* i= 2:   -0.277754787390380   -4.840013326416516 *
*************************************************

BRAESS - iteration step 8
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.401531034759345   +0.925987206939884 *
* i= 2:   -0.367831913059124  -11.274807455871752 *
*************************************************

BRAESS - iteration step 9
Factor C=1.000000 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.457517368875552   +0.803059932383429 *
* i= 2:   -0.427195133713197   -9.058735176928465 *
*************************************************
```

```
BRAESS - iteration step 10
Factor C=1.000000 yields better approximation:
*****************************************************
* BRAESS-iteration terminated with approximation: *
*                   a[i]                  t[i]      *
*          ------------------   ------------------ *
* i= 1:   +0.460971153210493   +0.803898614661879 *
* i= 2:   -0.431091336780594   -9.445104990768924 *
*****************************************************

                END OF Braess ITERATION:
          a best approximation has been calculated
                with delta_B = 1.000000E-02
```



### 3.16.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-01/t-01step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.278887720646740   +1.417992614795477
  i= 2:   -0.084232105112638   -1.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.419356907082145:
           x                  err(x)
  ------------------     ------------------
  +0.419356907082145     +0.197505163901540 (*)
  +0.000000000000000     -0.194655615534102
  +1.000000000000000     -0.120491047798323
A zero has been found at   x=  +0.047527698606551
A zero has been found at   x=  +0.881944643065345
```

### 3.16.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 10:**

```
Input from job file "../Jobs/para72/eval-t-01":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.460971153210493   +0.803898614661879
  i= 2:   -0.431091336780594   -9.445104990768924
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029879816429899
i= 1:   +0.016894143490602   +0.030214000410553
i= 2:   +0.178113814432268   -0.029740138596422
i= 3:   +0.634923910676457   +0.029923566807969
i= 4:   +1.000000000000000   -0.029883514547702

Norm of error function:       0.030214000410553
Relative deviation:           0.015683517829226


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001254893309151
i= 1:   +0.069130678341164
i= 2:   +0.366466586900239
i= 3:   +0.893216695870559
```

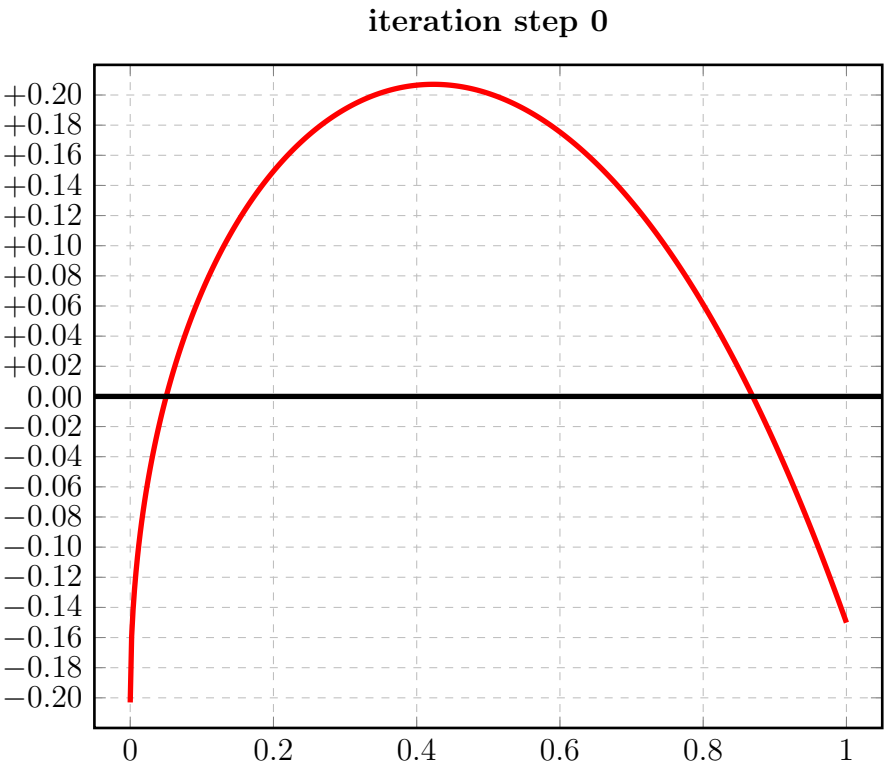
## 3.17 Berechnung für $t_2 = 0.0$

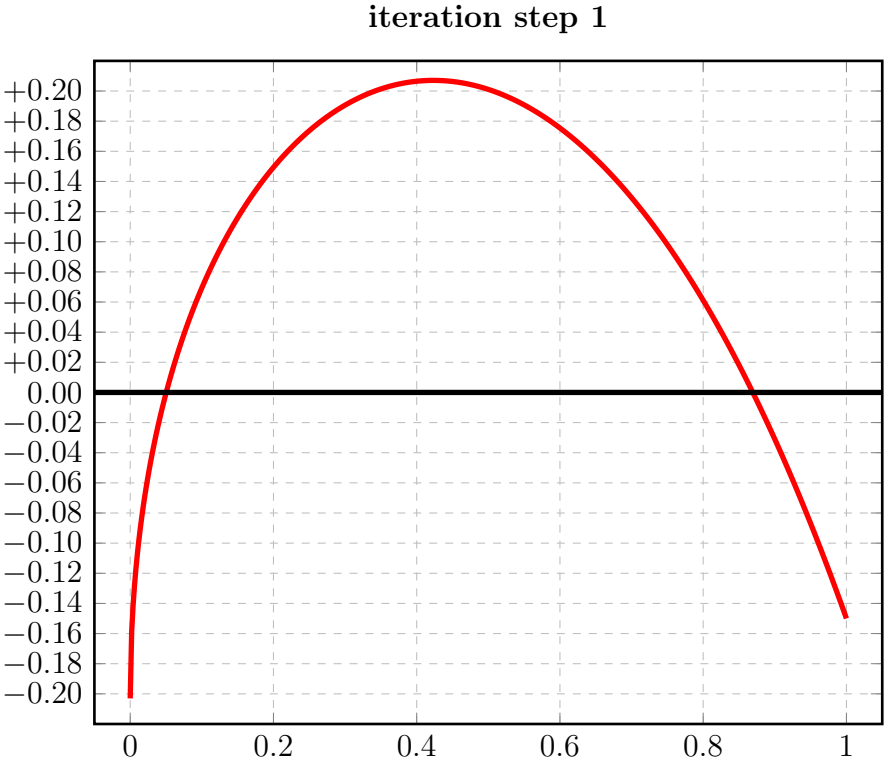
### 3.17.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt72/abschnitt72-t-00-terse-plots":
------------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                 a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   -0.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor C=0.062500=1/(2**4) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.269067719794642   +1.508455828765336 *
* i= 2:   -0.066009571702311   +0.000000000000000 *
*************************************************

BRAESS - iteration step 1
Factor C=0.000244=1/(2**12) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.267465594506113   +1.511334349544726 *
* i= 2:   -0.064444207620922   -0.032591958287888 *
*************************************************
```

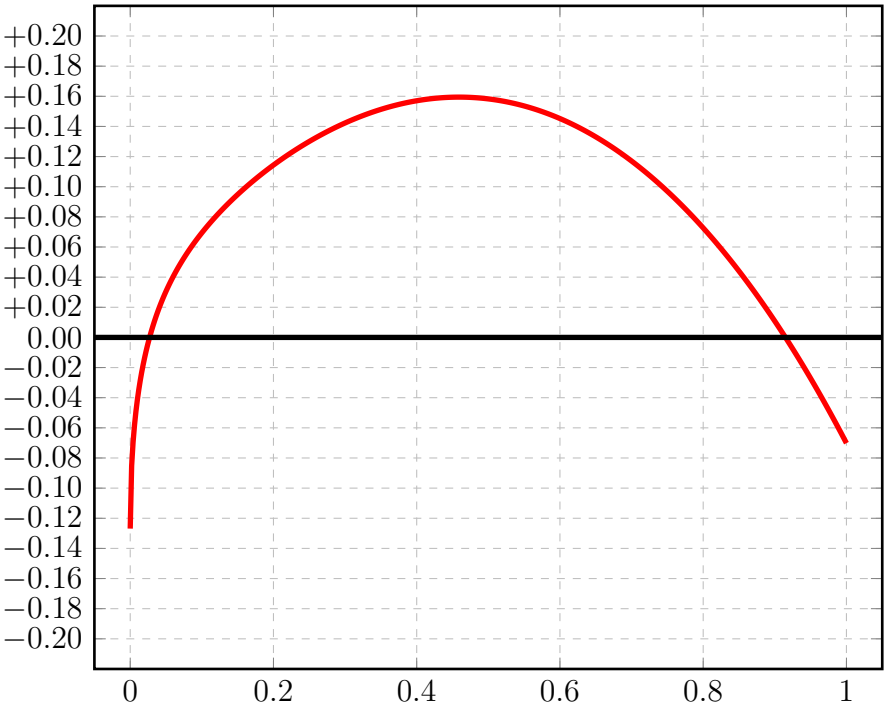
```
-------
OMITTED: iteration steps 2 - 17
-------

BRAESS - iteration step 18
Factor C=0.250000=1/(2**2) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.286672449500455   +1.317385065819248 *
* i= 2:   -0.159808608078471   -6.893544438110808 *
*************************************************

BRAESS - iteration step 19
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.424240857573584   +0.799381768454104 *
* i= 2:   -0.393851735019205  -14.665854543261224 *
*************************************************

BRAESS - iteration step 20
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.449546399072895   +0.835615989054799 *
* i= 2:   -0.415492194558062   -7.879243756820243 *
*************************************************
```

```
BRAESS - iteration step 21
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.458496845251566   +0.809403756753102 *
* i= 2:   -0.428192165202850   -9.414277146033191 *
*************************************************

BRAESS - iteration step 22
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.461132381121110   +0.803481111443147 *
* i= 2:   -0.431301529363997   -9.452123123897627 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

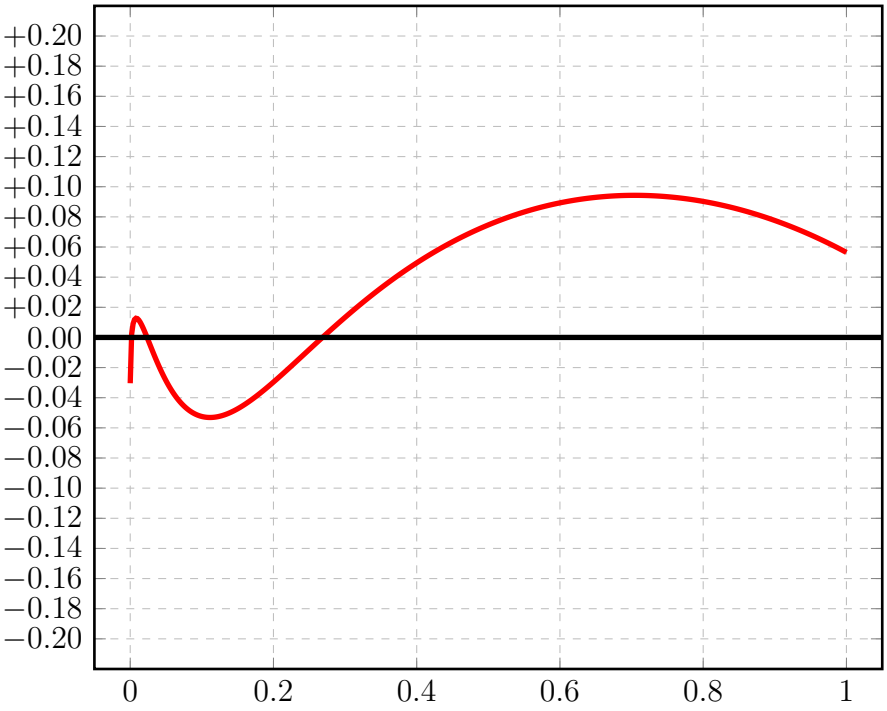

iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

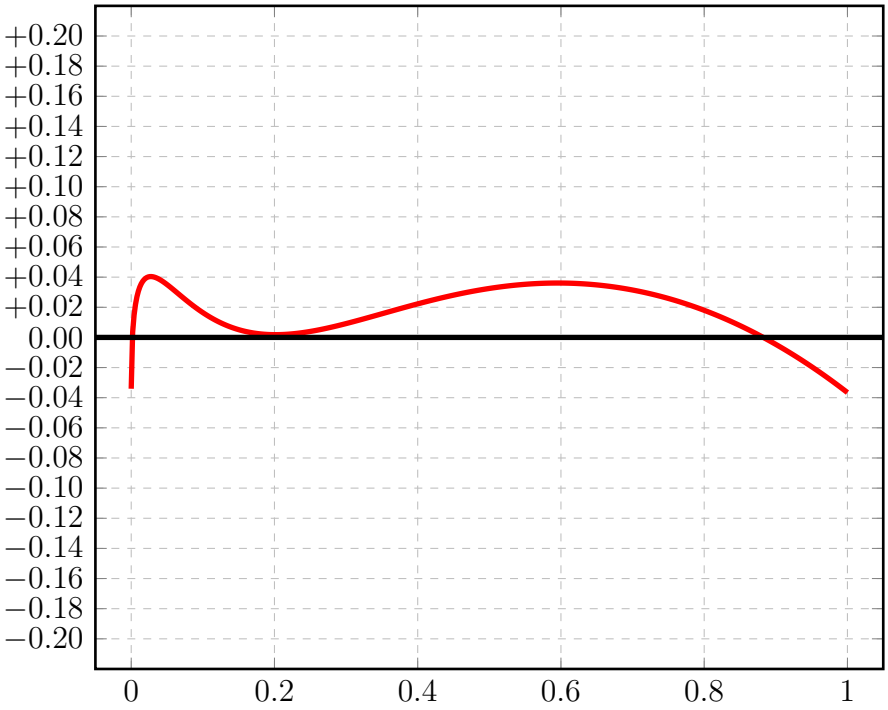
**iteration step 18**



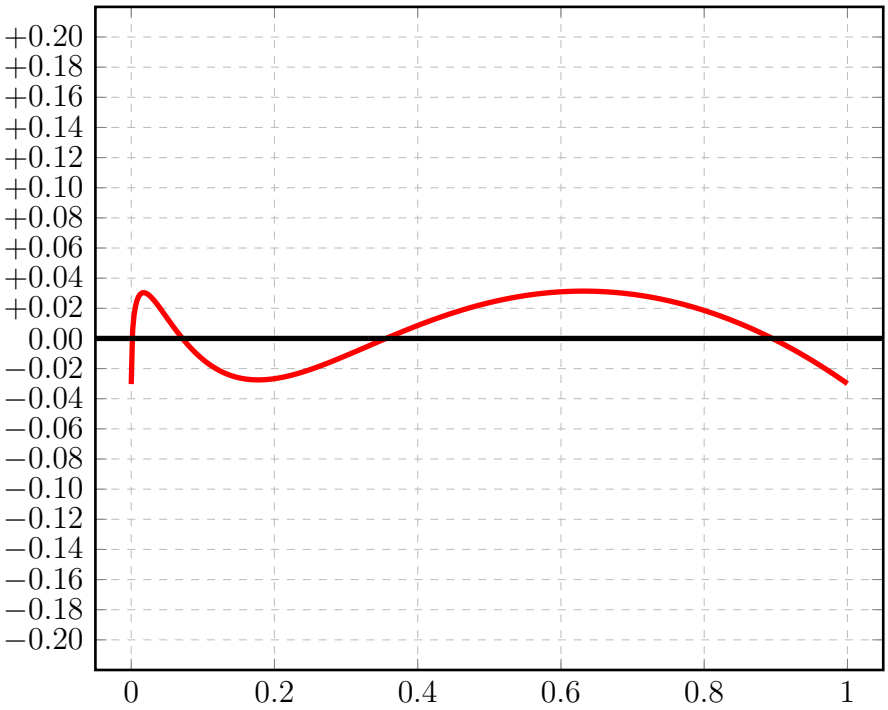

**iteration step 19**

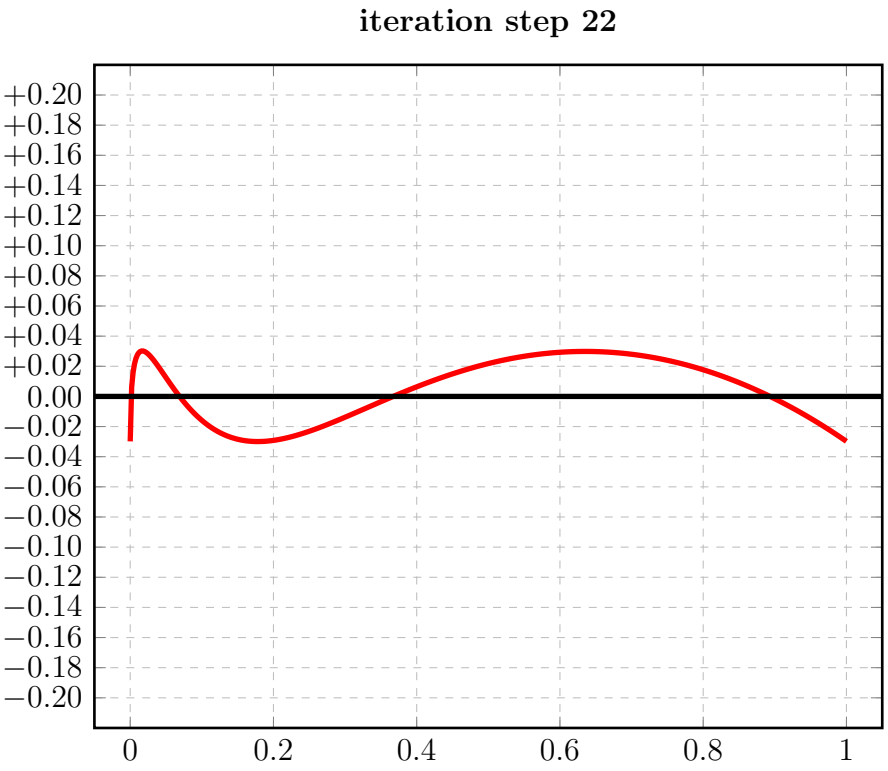


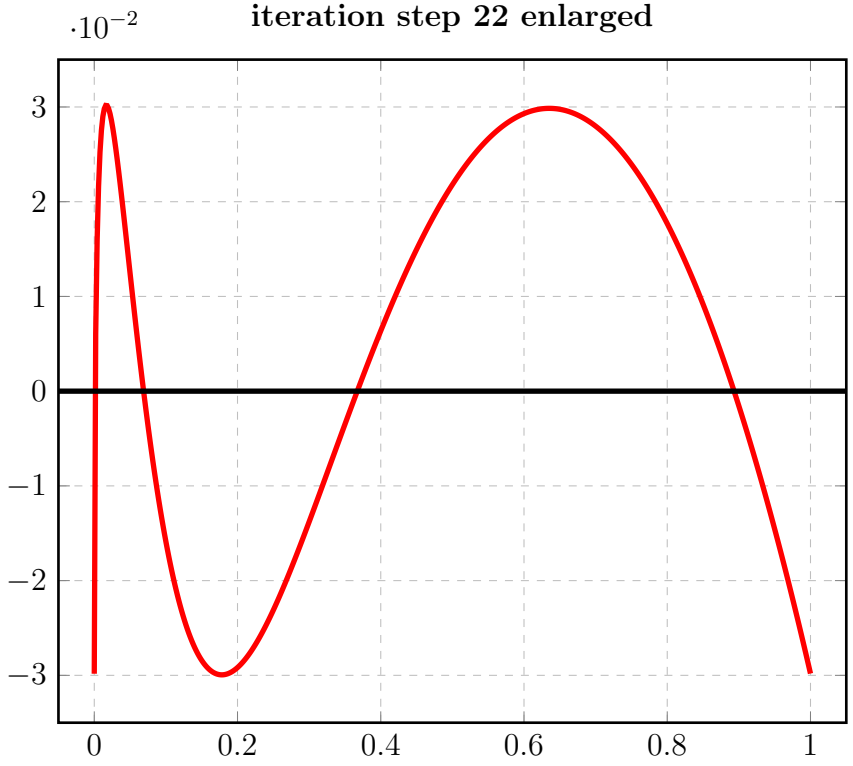
**iteration step 20**




**iteration step 21**

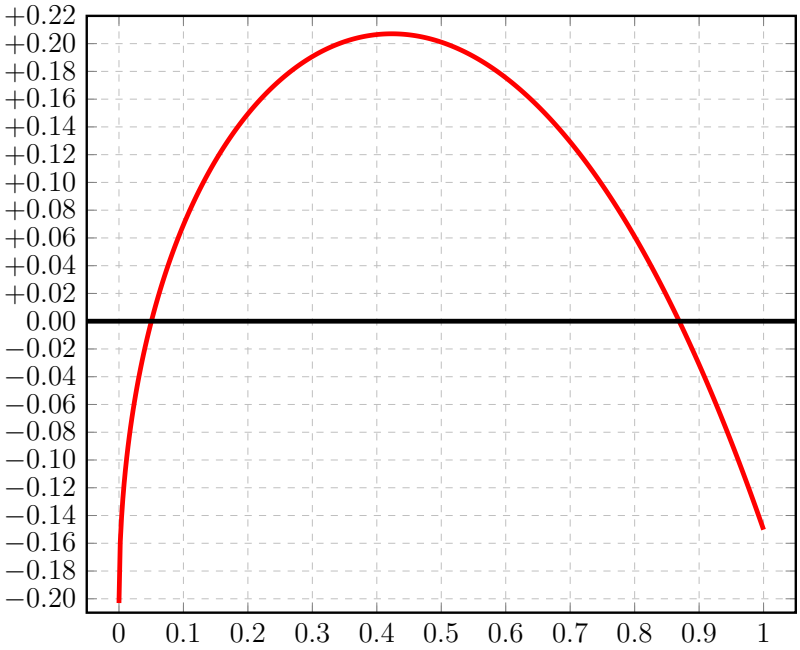



iteration step 22
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

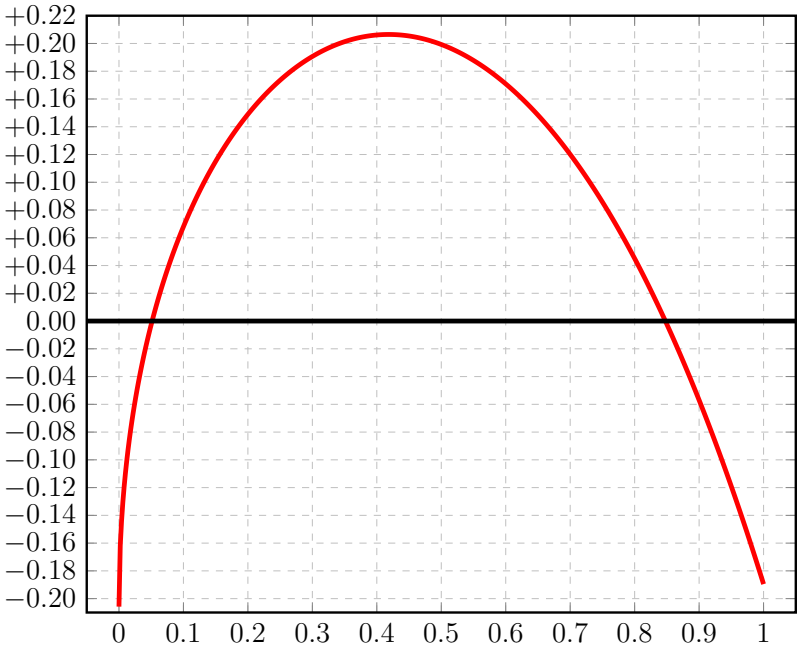

iteration step 22 enlarged
$\cdot 10^{-2}$
3
2
1
0
−1
−2
−3
0
0.2
0.4
0.6
0.8
1

Zum Vergleich: die Startfunktionen von [1] nach Fall 2 (Zeichnung 14C) und Fall 1 (Zeichnung 14H):

**Zeichnung 14C von [1], §7.2**




**Zeichnung 14H von [1], §7.2**



### 3.17.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t-00/t-00step0_x":
-----------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.269067719794642   +1.508455828765336
  i= 2:   -0.066009571702311   +0.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.423250262431928:
           x                  err(x)
  ------------------     ------------------
  +0.423250262431928     +0.207092775191915 (*)
  +0.000000000000000     -0.203058148092331
  +1.000000000000000     -0.150108216499894
A zero has been found at   x=  +0.050295323231743
A zero has been found at   x=  +0.869158296866928
```

### 3.17.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 22:**

```
   Input from job file "../Jobs/para72/eval-t-00":
 ------------------------------------------------
 - Function: f(x)=sqrt(x)
 - Approximation with respect to V_2
 - Interval I                    : [0.00,1.00]
 - Distance of equidistant points: 0.0010
 - The parameters of approximation:
                 a[i]                 t[i]
   i= 1:   +0.461132381121110   +0.803481111443147
   i= 2:   -0.431301529363997   -9.452123123897627
 - output: terse
 ------------------------------------------------

 The exact local extrema of f-E(a) in interval I:
 ------------------------------------------------
               x[i]                y[i]
         ------------------   ------------------
 i= 0:   +0.000000000000000   -0.029830851757113
 i= 1:   +0.016847604690215   +0.030189552435806
 i= 2:   +0.178079585175653   -0.029947723130137
 i= 3:   +0.635260391759564   +0.029854389140408
 i= 4:   +1.000000000000000   -0.029813902860873

 Norm of error function:       0.030189552435806
 Relative deviation:           0.012443032261984


 The exact zeroes of f-E(a) in interval I:
 -----------------------------------------
               x[i]
         ------------------
 i= 0:   +0.001250491947217
 i= 1:   +0.068875888460633
 i= 2:   +0.367213112364403
 i= 3:   +0.893309271063014
```

## 3.18 Berechnung für $t_2 = +1.0$

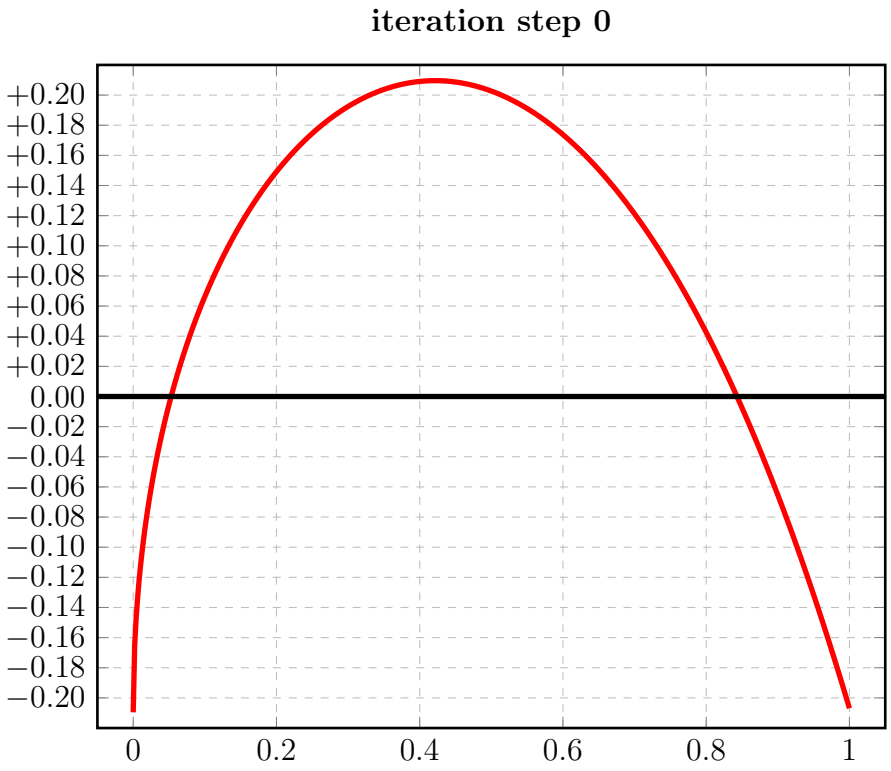
### 3.18.1 Die Berechnung

```
  Input from job file "../Jobs/abschnitt72/abschnitt72-t+01-terse":
  ----------------------------------------------------------------
  - Function: f(x)=sqrt(x)
  - Approximation with respect to V_2
  - Interval                       : [0.00,1.00]
  - Distance of equidistant points: 0.0100
  - Braess termination criterion  : 1.000000E-02
  - Remez  termination criterion  : 1.000000E-02
  - Plot-Indicator                 : 0
  - output                         : terse
  - Starting parameters:
                  a[i]                 t[i]
    i= 1:   +0.209953239200000   +1.751452157000000
    i= 2:   +0.000000000000000   +1.000000000000000
  ------------  End Of Initialization  -----------

 BRAESS - iteration step 0
  Factor C=0.003906=1/(2**8) yields better approximation:
  ***************************************************
  * BRAESS-iteration terminated with approximation: *
  *                   a[i]                 t[i]     *
  *         ------------------   ------------------ *
  * i= 1:   +0.223074754575779   +1.718255982726203 *
  * i= 2:   -0.013504838259559   +1.000000000000000 *
  ***************************************************

  BRAESS - iteration step 1
  Factor C=0.000002=1/(2**19) yields better approximation:
  ***************************************************
  * BRAESS-iteration terminated with approximation: *
  *                   a[i]                 t[i]     *
  *         ------------------   ------------------ *
  * i= 1:   +0.222963036479692   +1.718399948981899 *
  * i= 2:   -0.013393403357286   +0.996170513889157 *
  ***************************************************
```

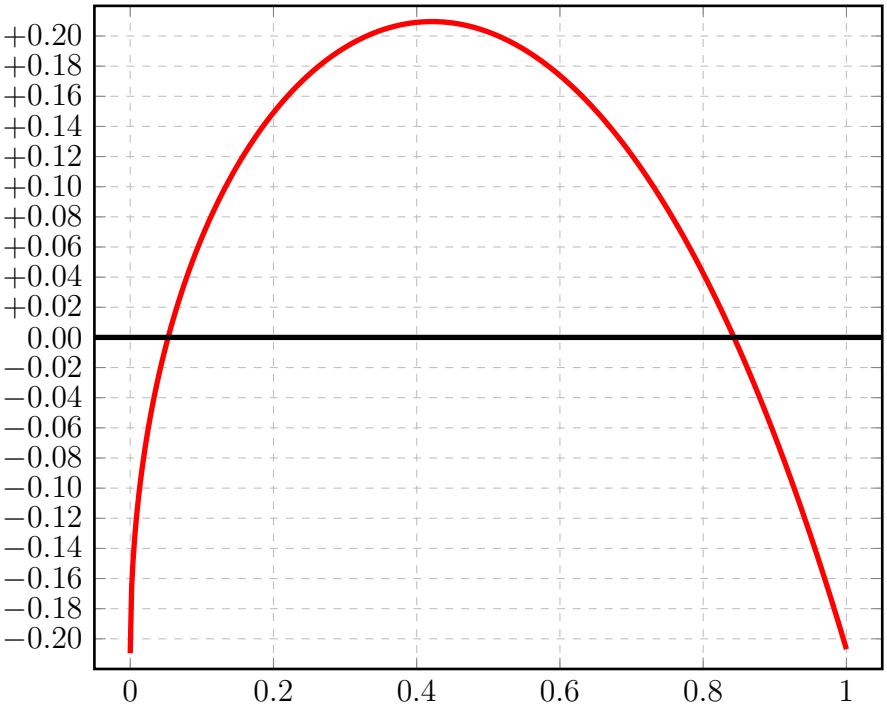
```
BRAESS - iteration step 2
Factor C=0.000002=1/(2**19) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.222853129647993   +1.718542269665509 *
* i= 2:   -0.013283779774890   +0.992345002494669 *
*************************************************

-------
OMITTED: iteration steps 3-89
-------

BRAESS - iteration step 90
Factor C=0.007812=1/(2**7) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211920698845608   +1.736804080398448 *
* i= 2:   -0.004189475636014  -11.087710235674738 *
*************************************************

BRAESS - iteration step 91
Factor C=0.500000=1/(2**1) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.301653618884709   +1.239640081800617 *
* i= 2:   -0.184765443619702  -75.808987979908125 *
*************************************************
```

```
BRAESS - iteration step 92
Factor C=0.250000=1/(2**2) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.312584371726683   +1.215465931702107 *
* i= 2:   -0.201843602159048  -11.382593901146620 *
*************************************************

BRAESS - iteration step 93
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.442044111401594   +0.777725588803842 *
* i= 2:   -0.412917193121023   -8.590863822715146 *
*************************************************

BRAESS - iteration step 94
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.460345883463022   +0.806877375019314 *
* i= 2:   -0.430361530332815   -9.458571433169160 *
*************************************************
```

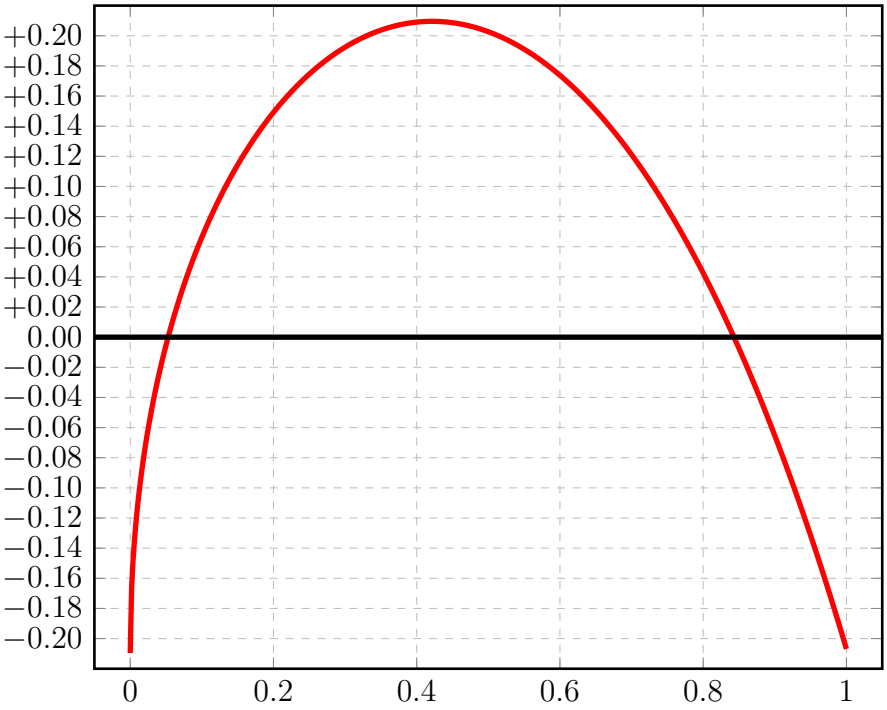
```
BRAESS - iteration step 95
Factor C=1.000000=1/(2**0) yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                   a[i]                  t[i]      *
*          ------------------   ------------------ *
* i= 1:   +0.461136579423758   +0.803488299666518 *
* i= 2:   -0.431305830707916   -9.451791745076736 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```



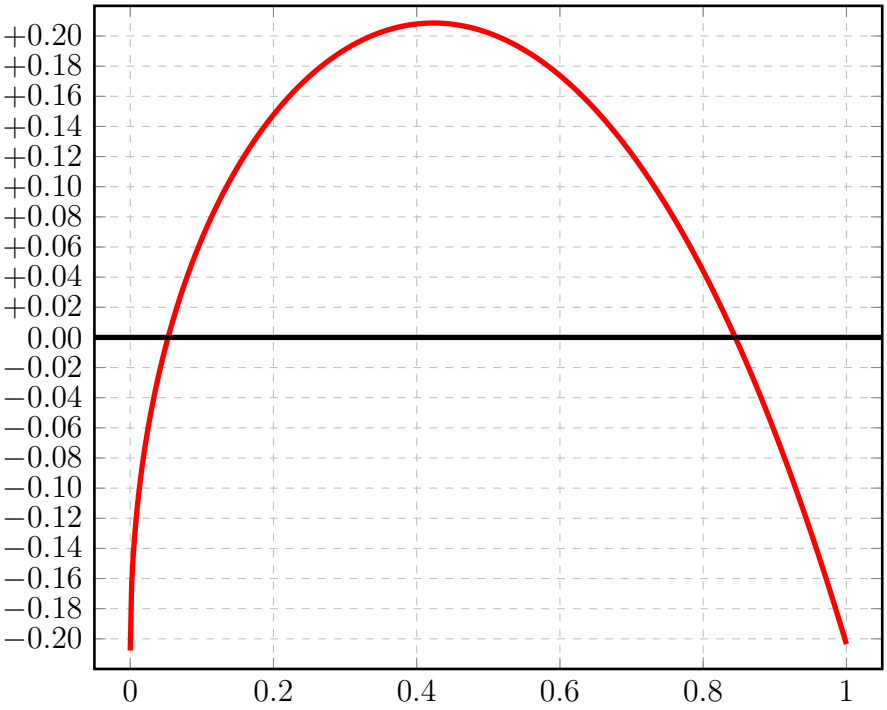
**iteration step 1**



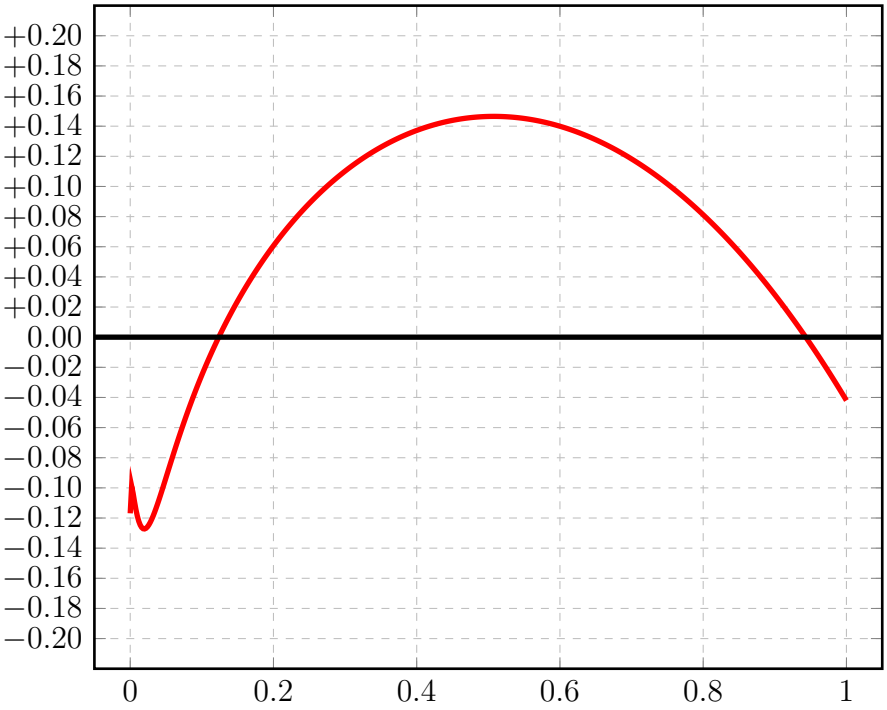

**iteration step 2**



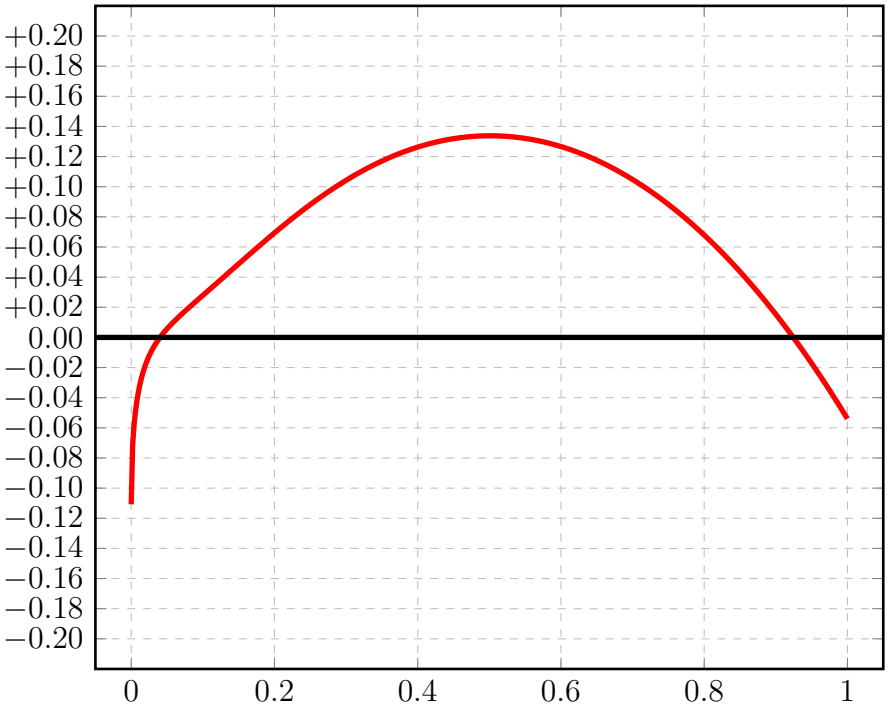
**iteration step 90**



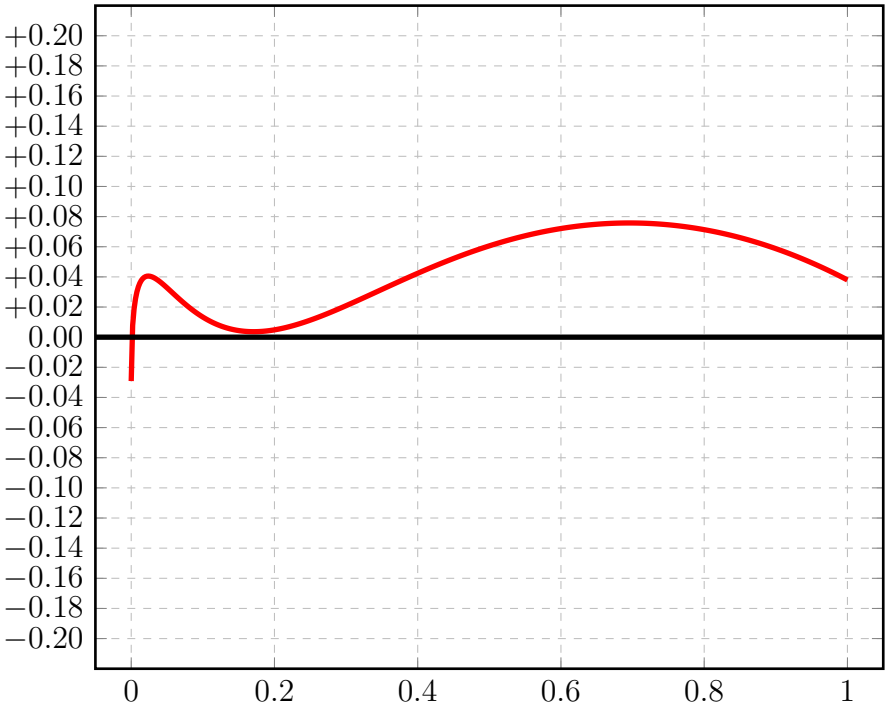

**iteration step 91**



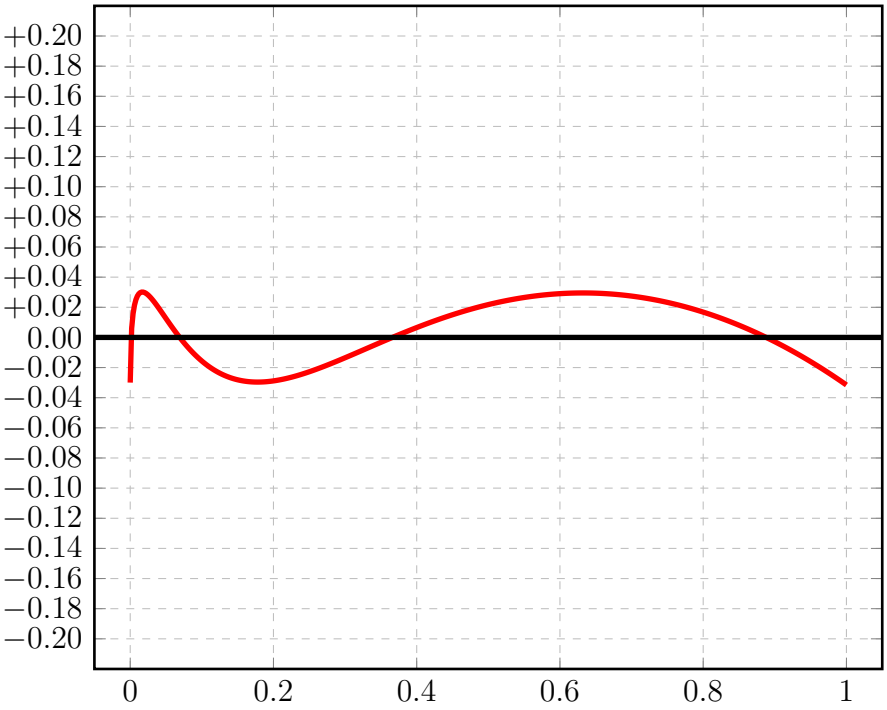
**iteration step 92**



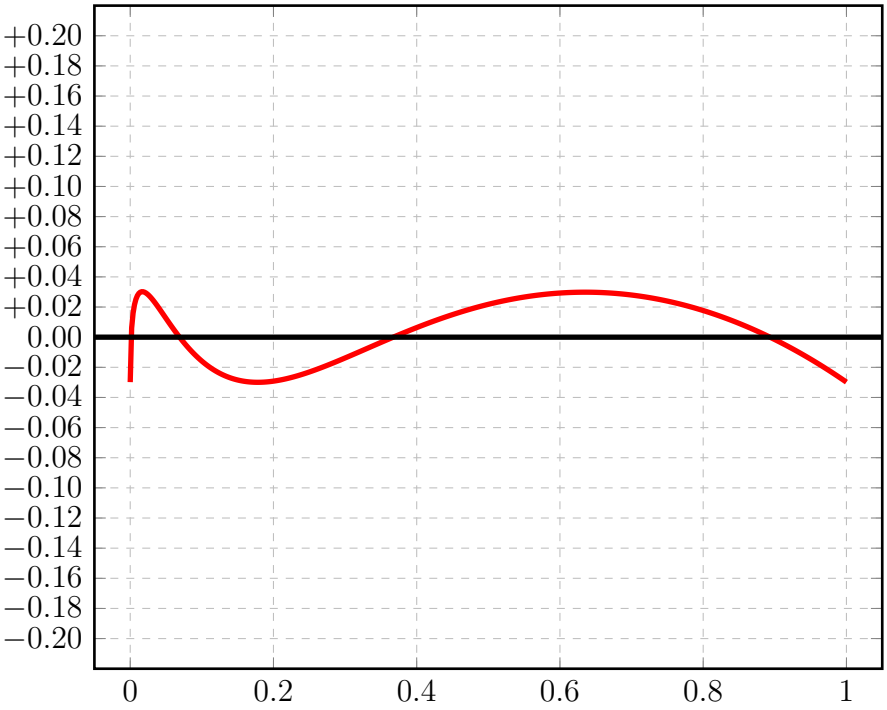

**iteration step 93**

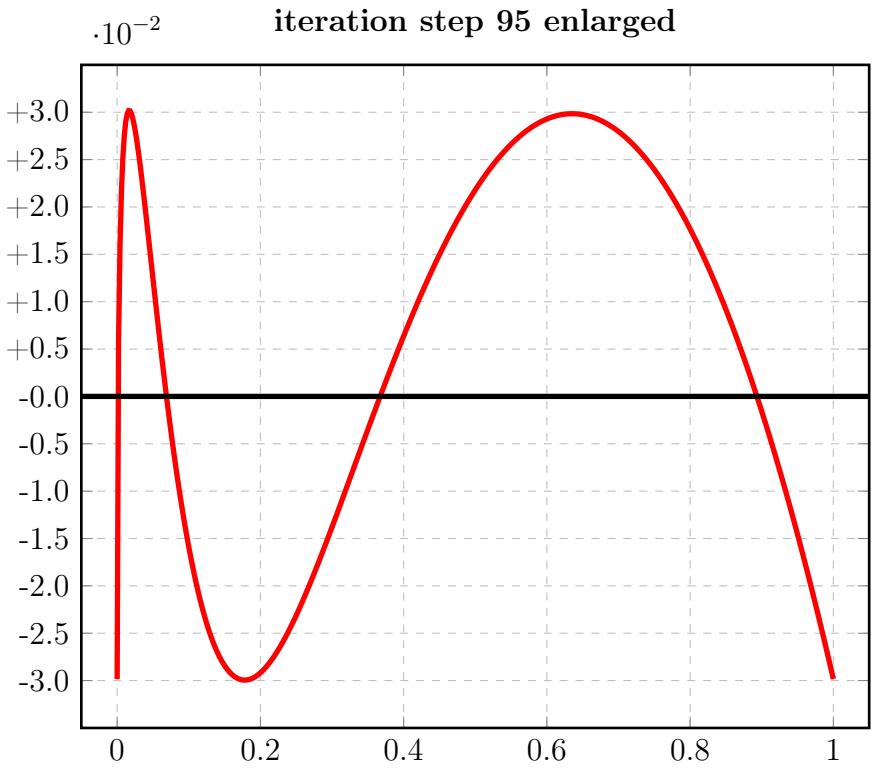


**iteration step 94**




**iteration step 95**




iteration step 95 enlarged
$\cdot 10^{-2}$
+3.0
+2.5
+2.0
+1.5
+1.0
+0.5
-0.0
-0.5
-1.0
-1.5
-2.0
-2.5
-3.0
0
0.2
0.4
0.6
0.8
1

### 3.18.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+01/t+01step0_x":
-----------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.223074754575779   +1.718255982726203
  i= 2:   -0.013504838259559   +1.000000000000000
- Action: find local extremum near x=+0.000000
- output: terse
-----------------------------------------------------

A local extremum has been found at x=  +0.421461953858259:
           x                  err(x)
  ------------------     ------------------
  +0.421461953858259     +0.209573291590398 (*)
  +0.000000000000000     -0.209569916316220
  +1.000000000000000     -0.206886613893029
A zero has been found at   x=  +0.052935585282514
A zero has been found at   x=  +0.842970017980982
```

### 3.18.3 Auswertungen mit EXPAPP_EVAL

**Auswertung Iteration Step 95:**

```
Input from job file "../Jobs/para72/eval-t+01":
-------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval I                    : [0.00,1.00]
- Distance of equidistant points: 0.0010
- The parameters of approximation:
                 a[i]                 t[i]
  i= 1:   +0.461136579423758   +0.803488299666518
  i= 2:   -0.431305830707916   -9.451791745076736
- output: terse
-------------------------------------------------

The exact local extrema of f-E(a) in interval I:
-------------------------------------------------
               x[i]                 y[i]
        ------------------   ------------------
i= 0:   +0.000000000000000   -0.029830748715842
i= 1:   +0.016848350513707   +0.030190961938546
i= 2:   +0.178089320584655   -0.029947720969091
i= 3:   +0.635242701588068   +0.029844121484733
i= 4:   +1.000000000000000   -0.029830670244967

Norm of error function:       0.030190961938546
Relative deviation:           0.011933759987915


The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.001250469281772
i= 1:   +0.068880294661217
i= 2:   +0.367240562609641
i= 3:   +0.893245955699556
```

## 3.19 Berechnung für $t_2 = +2.0$

### 3.19.1 Die Berechnung

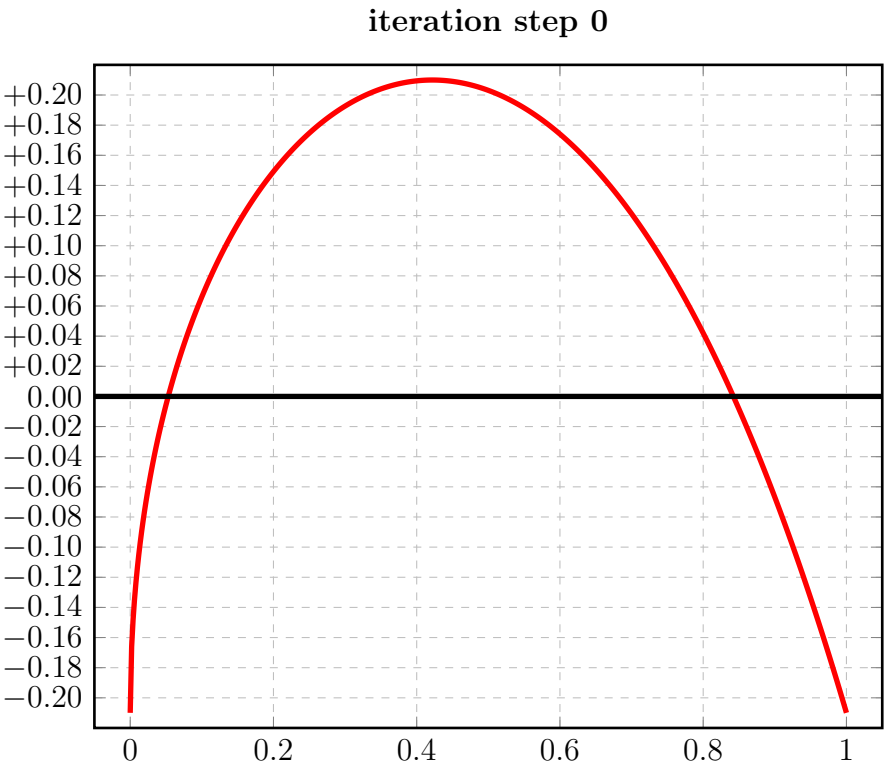
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+02-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +2.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211049517439881   +1.752940821010503 *
* i= 2:   -0.001101546410181   +2.000000000000000 *
*************************************************
```

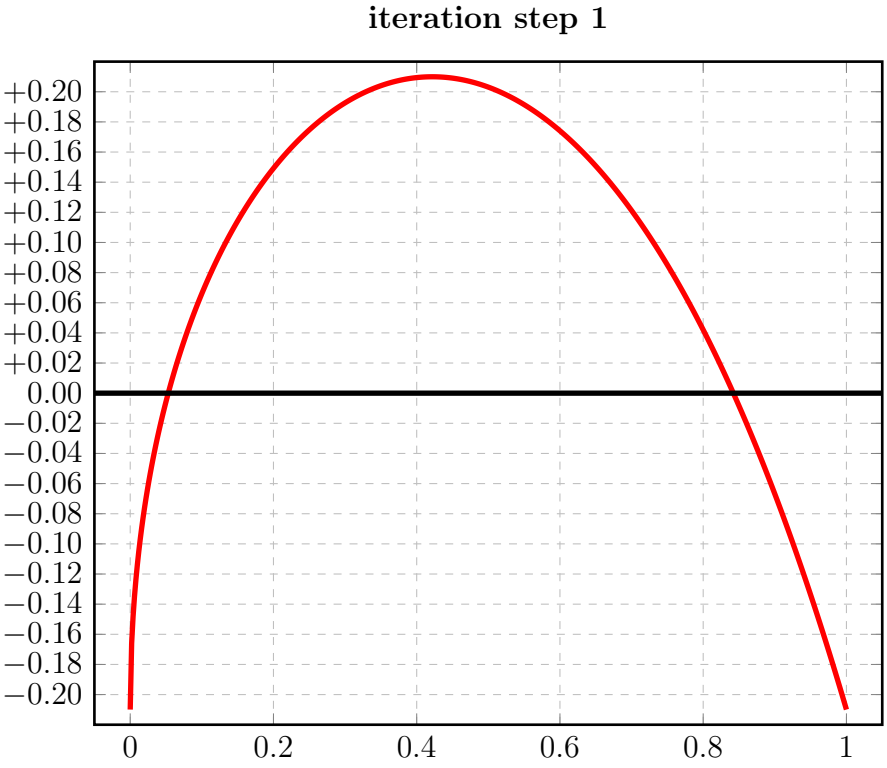
```
BRAESS - iteration step 1
Factor c=5.96e-08=2^-24 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211126225777528   +1.752989599339092 *
* i= 2:   -0.001178263074823   +1.992049367124329 *
*************************************************

-------
OMITTED: iteration steps 2 - 99
-------

BRAESS - iteration step 100
Factor c=5.82e-11=2^-34 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211152728076317   +1.753005780171301 *
* i= 2:   -0.001204767944845   +1.989590443466733 *
*************************************************

-------
OMITTED: iteration steps 101 - 199
-------

BRAESS - iteration step 200
Factor c=5.82e-11=2^-34 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211161316210533   +1.753010979114909 *
* i= 2:   -0.001213356892845   +1.988812469405126 *
*************************************************
```

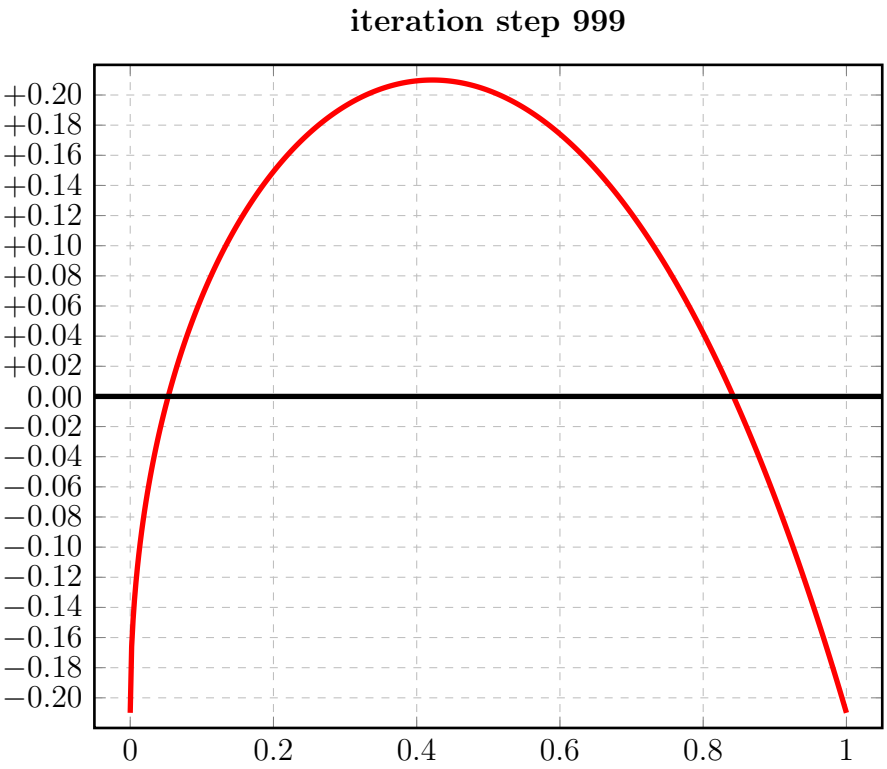
```
-------
OMITTED: iteration steps 201 - 299
-------

BRAESS - iteration step 300
Factor c=5.82e-11=2^-34 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211169991450511   +1.753016211757293 *
* i= 2:   -0.001222032946651   +1.988034573790546 *
**************************************************

-------
OMITTED: iteration steps 301 - 399
-------

BRAESS - iteration step 400
Factor c=5.82e-11=2^-34 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211178754965021   +1.753021478429169 *
* i= 2:   -0.001230797275031   +1.987256756253404 *
**************************************************

-------
OMITTED: iteration steps 401 - 499
-------

BRAESS - iteration step 500
Factor c=5.82e-11=2^-34 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211187607942495   +1.753026779465629 *
* i= 2:   -0.001239651066419   +1.986479016419741 *
**************************************************
```

```
-------
OMITTED: iteration steps 501 - 599
-------

BRAESS - iteration step 600
Factor c=5.82e-11=2^-34 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211196551591430   +1.753032115206209 *
* i= 2:   -0.001248595529312   +1.985701353911157 *
*************************************************

-------
OMITTED: iteration steps 601 - 999
-------

BRAESS - iteration step 700
Factor c=5.82e-11=2^-34 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211205587140799   +1.753037485994965 *
* i= 2:   -0.001257631892682   +1.984923768344732 *
*************************************************

-------
OMITTED: iteration steps 701 - 799
-------

BRAESS - iteration step 800
Factor c=5.82e-11=2^-34 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211214715840466   +1.753042892180549 *
* i= 2:   -0.001266761406393   +1.984146259332955 *
*************************************************
```

```
-------
OMITTED: iteration steps 801 - 899
-------

BRAESS - iteration step 900
Factor c=5.82e-11=2^-34 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211223938961618   +1.753048334116287 *
* i= 2:   -0.001275985341632   +1.983368826483645 *
**************************************************

-------
OMITTED: iteration steps 901 - 998
-------

BRAESS - iteration step 999
Factor c=5.82e-11=2^-34 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211233164130760   +1.753053757199892 *
* i= 2:   -0.001285211316763   +1.982599242596980 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.19.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+02/t+02step0_x":
A local extremum has been found at x=  +0.421824863712642:
           x                  err(x)
  ------------------     ------------------
  +0.421824863712642     +0.209945480139769
  +0.000000000000000     -0.209947971029700 (*)
  +1.000000000000000     -0.209943631654013
A zero has been found at   x=  +0.053087673564892
A zero has been found at   x=  +0.842125904704790
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+02/t+02step1_x":
A local extremum has been found at x=  +0.421823741483345:
           x                  err(x)
  ------------------     ------------------
  +0.421823741483345     +0.209944877657433
  +0.000000000000000     -0.209947962702705
  +1.000000000000000     -0.209947878853414
A zero has been found at   x=  +0.053087691796980
A zero has been found at   x=  +0.842123423062282
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+02/t+02step999_x":
A local extremum has been found at x=  +0.421823711863842:
           x                  err(x)
  ------------------     ------------------
  +0.421823711863842     +0.209944846074822
  +0.000000000000000     -0.209947952813997
  +1.000000000000000     -0.209947957567539 (*)
A zero has been found at   x=  +0.053087689604059
A zero has been found at   x=  +0.842123375832825
```

## 3.20 Berechnung für $t_2 = +3.0$

### 3.20.1 Die Berechnung

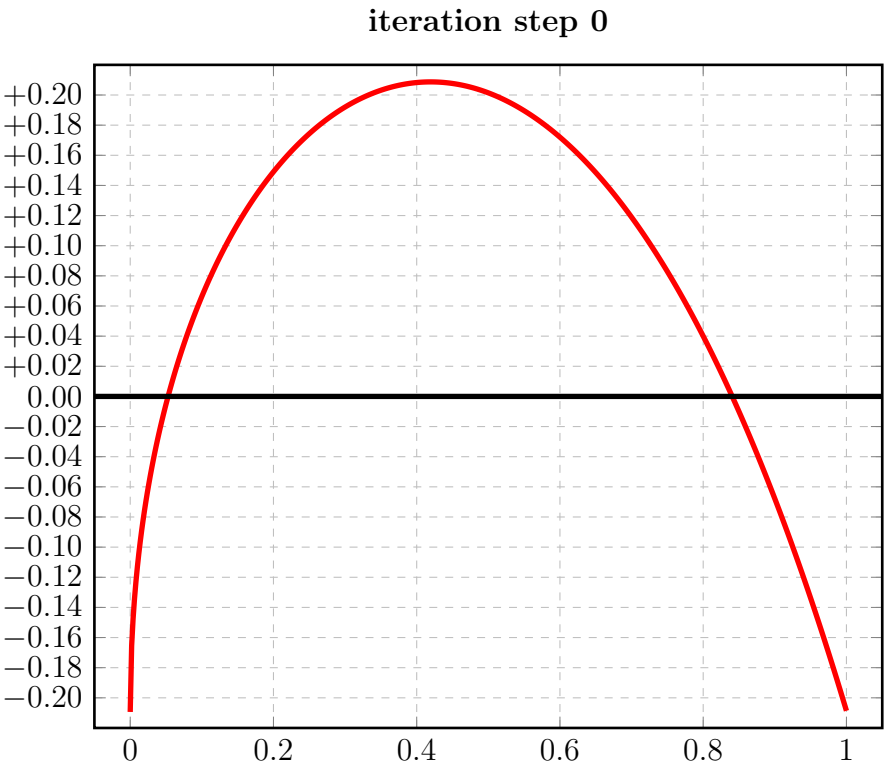
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+03-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +3.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=7.81e-03=2^-7 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.212398891466741   +1.787989634234983 *
* i= 2:   -0.003036632172054   +3.000000000000000 *
**************************************************
```

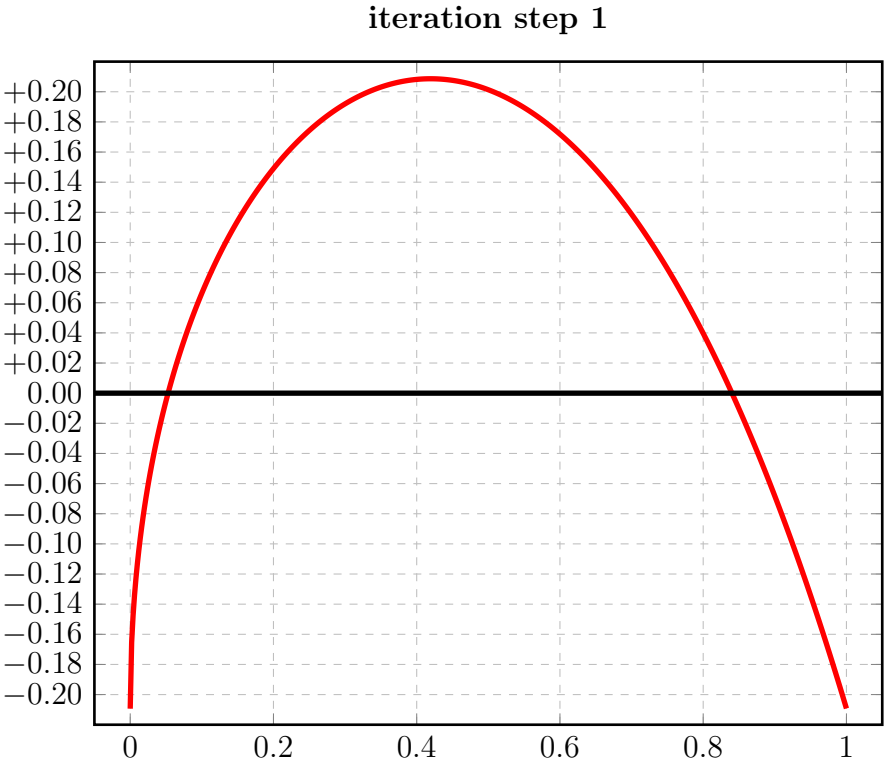
```
BRAESS - iteration step 1
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.212930343775655   +1.790376110088441 *
* i= 2:   -0.003575954177633   +2.924282884308652 *
*************************************************

-------
OMITTED: iteration steps 2 - 99
-------

BRAESS - iteration step 100
Factor c=2.38e-07=2^-22 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.213821766037662   +1.793778240162270 *
* i= 2:   -0.004476771707392   +2.834155844234593 *
*************************************************

-------
OMITTED: iteration steps 101 - 199
-------

BRAESS - iteration step 200
Factor c=2.38e-07=2^-22 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.214182106516617   +1.795032818144317 *
* i= 2:   -0.004840228029778   +2.804582593260866 *
*************************************************
```

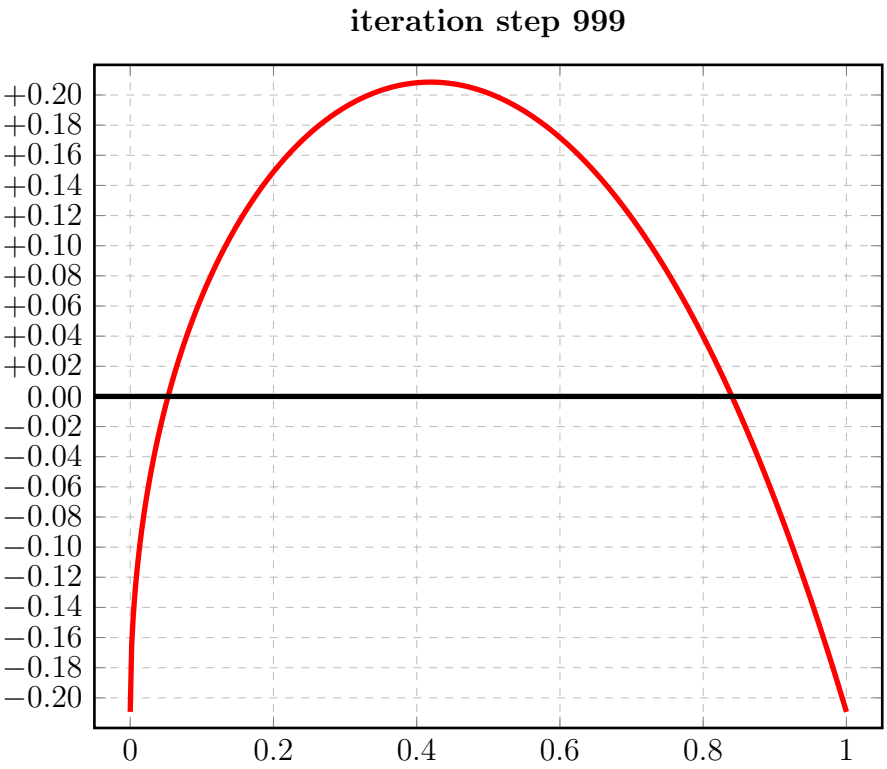
```
-------
OMITTED: iteration steps 201 - 299
-------

BRAESS - iteration step 300
Factor c=2.38e-07=2^-22 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.214579975312310   +1.796357147563959 *
* i= 2:   -0.005241219561874   +2.775092659205593 *
*************************************************

-------
OMITTED: iteration steps 301 - 399
-------

BRAESS - iteration step 400
Factor c=2.38e-07=2^-22 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.215020523227408   +1.797757594211709 *
* i= 2:   -0.005684896603824   +2.745674914167298 *
*************************************************

-------
OMITTED: iteration steps 401 - 499
-------

BRAESS - iteration step 500
Factor c=2.38e-07=2^-22 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.215509778727345   +1.799241234819822 *
* i= 2:   -0.006177287615453   +2.716320508906087 *
*************************************************
```

```
-------
OMITTED: iteration steps 501 - 599
-------

BRAESS - iteration step 600
Factor c=1.19e-07=2^-23 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.215989079443499   +1.800629995539173 *
* i= 2:   -0.006659368532809   +2.690389004953066 *
**************************************************

-------
OMITTED: iteration steps 601 - 699
-------

BRAESS - iteration step 700
Factor c=1.19e-07=2^-23 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.216281418969939   +1.801448587406996 *
* i= 2:   -0.006953280982192   +2.675755461112572 *
**************************************************

-------
OMITTED: iteration steps 701 - 799
-------

BRAESS - iteration step 800
Factor c=1.19e-07=2^-23 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.216590742424360   +1.802293183540246 *
* i= 2:   -0.007264178933571   +2.661132414572780 *
**************************************************
```

```
-------
OMITTED: iteration steps 801 - 899
-------

BRAESS - iteration step 900
Factor c=1.19e-07=2^-23 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.216918357963476   +1.803165117110440 *
* i= 2:   -0.007593370542782   +2.646518316050462 *
*                                                 *
**************************************************

-------
OMITTED: iteration steps 901 - 998
-------

BRAESS - iteration step 999
Factor c=1.19e-07=2^-23 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.217262129038960   +1.804056665572370 *
* i= 2:   -0.007938703476406   +2.632057556671184 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.20.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+03/t+03step0_x":
A local extremum has been found at x=  +0.419512441439806:
           x                  err(x)
  ------------------     ------------------
  +0.419512441439806     +0.208701379849484
  +0.000000000000000     -0.209362259294687 (*)
  +1.000000000000000     -0.208605752796450
A zero has been found at   x=  +0.052849334538836
A zero has been found at   x=  +0.840505431566928
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+03/t+03step1_x":
A local extremum has been found at x=  +0.419377838983225:
           x                  err(x)
  ------------------     ------------------
  +0.419377838983225     +0.208629646233391
  +0.000000000000000     -0.209354389598022
  +1.000000000000000     -0.209228559194360
A zero has been found at   x=  +0.052848425148616
A zero has been found at   x=  +0.840180010846938
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+03/t+03step999_x":
A local extremum has been found at x=  +0.419315042685882:
           x                  err(x)
  ------------------     ------------------
  +0.419315042685882     +0.208553526093152
  +0.000000000000000     -0.209323425562554
  +1.000000000000000     -0.209335436245077 (*)
A zero has been found at   x=  +0.052839667931681
A zero has been found at   x=  +0.840123527061832
```

## 3.21 Berechnung für $t_2 = +4.0$

### 3.21.1 Die Berechnung

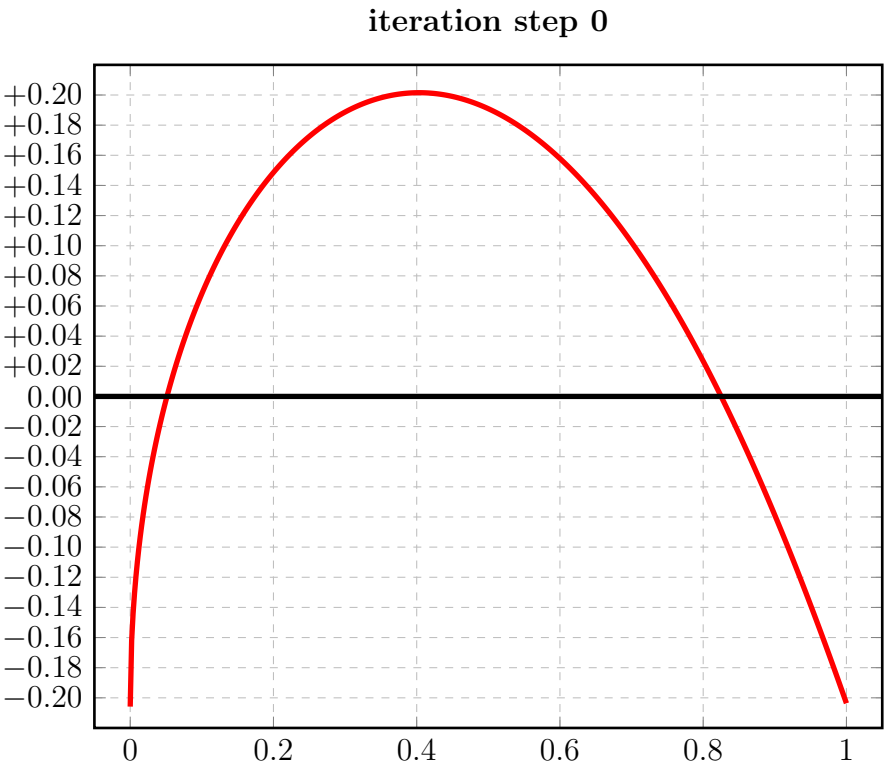
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+04-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +4.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=6.25e-02=2^-4 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209717277862435   +1.909908887357741 *
* i= 2:   -0.003891740832484   +4.000000000000000 *
**************************************************
```

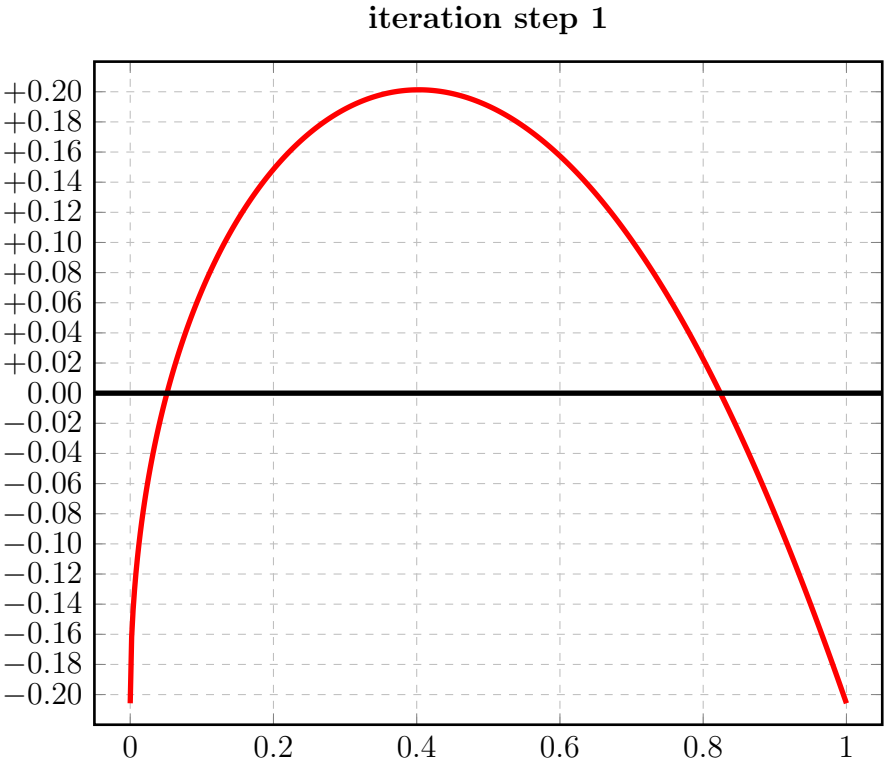
```
BRAESS - iteration step 1
Factor c=4.88e-04=2^-11 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.210284630575728   +1.916832485630407 *
* i= 2:   -0.004514494649067   +3.904409414257193 *
*************************************************

-------
OMITTED: iteration steps 2-99
-------

BRAESS - iteration step 100
Factor c=3.81e-06=2^-18 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.210861616691651   +1.922842788622808 *
* i= 2:   -0.005134775263881   +3.830359982726819 *
*************************************************

-------
OMITTED: iteration steps 101 - 199
-------

BRAESS - iteration step 200
Factor c=3.81e-06=2^-18 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211544700408339   +1.929346346872557 *
* i= 2:   -0.005861844305183   +3.755977138856855 *
*************************************************
```

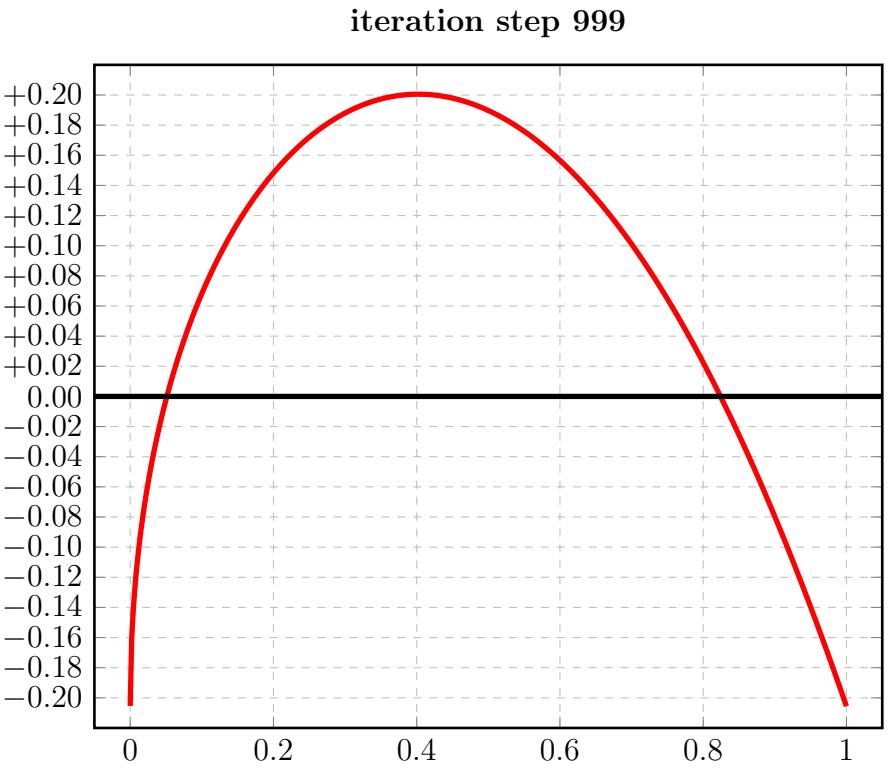
```
-------
OMITTED: iteration steps 201 - 299
-------

BRAESS - iteration step 300
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.212349206754678   +1.936337278485094 *
* i= 2:   -0.006710591307843   +3.681937505844866 *
**************************************************

-------
OMITTED: iteration steps 301 - 399
-------

BRAESS - iteration step 400
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.213301558892259   +1.943878429509408 *
* i= 2:   -0.007707422630116   +3.608153118443756 *
**************************************************

-------
OMITTED: iteration steps 401 - 499
-------

BRAESS - iteration step 500
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.214435775802099   +1.952044853500075 *
* i= 2:   -0.008886352776259   +3.534531440855829 *
**************************************************
```

```
-------
OMITTED: iteration steps 501 - 599
-------

BRAESS - iteration step 600
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.215798645803019   +1.960940810351893 *
* i= 2:   -0.010294200643008   +3.460861685900525 *
**************************************************

-------
OMITTED: iteration steps 601 - 699
-------

BRAESS - iteration step 700
Factor c=1.91e-06=2^-19 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.216868442398795   +1.967360403669169 *
* i= 2:   -0.011394266014275   +3.411461705926906 *
**************************************************

-------
OMITTED: iteration steps 701 - 799
-------

BRAESS - iteration step 800
Factor c=1.91e-06=2^-19 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.217764028793146   +1.972416690921872 *
* i= 2:   -0.012312516372346   +3.374567283473970 *
**************************************************
```

```
-------
OMITTED: iteration steps 801 - 899
-------

BRAESS - iteration step 900
Factor c=1.91e-06=2^-19 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.218755081519297   +1.977719844652547 *
* i= 2:   -0.013326306772230   +3.337632237035258 *
**************************************************

-------
OMITTED: iteration steps 901 - 998
-------

BRAESS - iteration step 999
Factor c=1.91e-06=2^-19 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.219844269342654   +1.983236482022350 *
* i= 2:   -0.014438055663767   +3.300984785480871 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.21.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+04/t+04step0_x":
A local extremum has been found at x=  +0.403365401302977:
           x                  err(x)
  ------------------     ------------------
  +0.403365401302977     +0.201530191803971
  +0.000000000000000     -0.205825537029951 (*)
  +1.000000000000000     -0.203628518693690
A zero has been found at   x=  +0.051318009519378
A zero has been found at   x=  +0.825037969288612
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+04/t+04step1_x":
A local extremum has been found at x=  +0.402963910965749:
           x                  err(x)
  ------------------     ------------------
  +0.402963910965749     +0.201303012463244
  +0.000000000000000     -0.205770135926661
  +1.000000000000000     -0.205793936249366
A zero has been found at   x=  +0.051301571520817
A zero has been found at   x=  +0.824041433974259
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+04/t+04step999_x":
A local extremum has been found at x=  +0.402249172320708:
           x                  err(x)
  ------------------     ------------------
  +0.402249172320708     +0.200527766633059
  +0.000000000000000     -0.205406213678887
  +1.000000000000000     -0.205597781352963 (*)
A zero has been found at   x=  +0.051182970026739
A zero has been found at   x=  +0.824162361265701
```

## 3.22 Berechnung für $t_2 = +5.0$

### 3.22.1 Die Berechnung

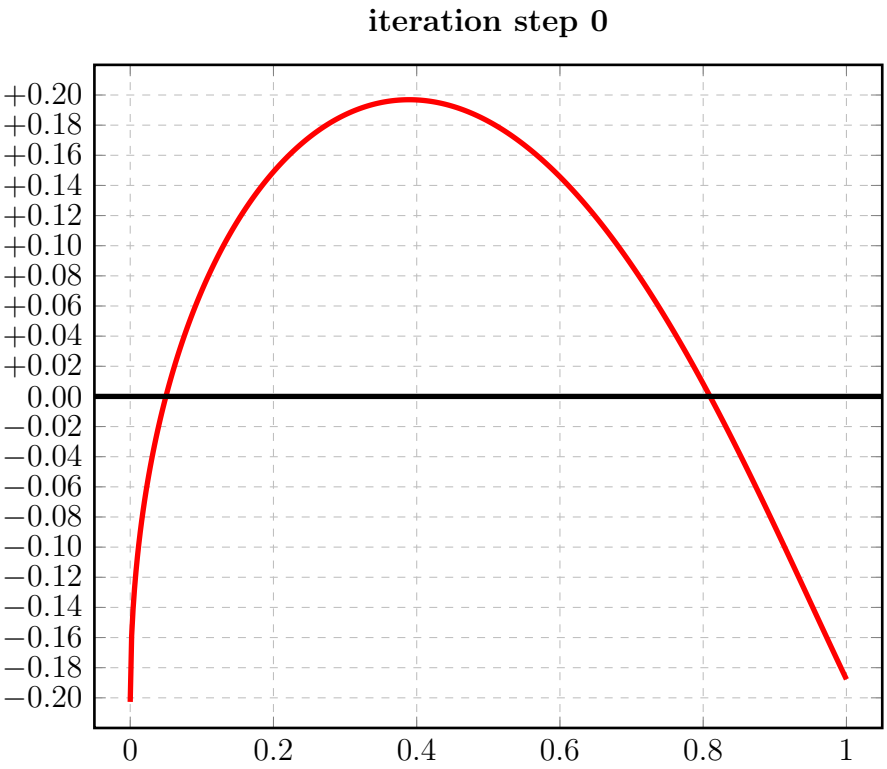
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+05-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +5.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=1.25e-01=2^-3 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.204558842616053   +1.967969113531704 *
* i= 2:   -0.001860489311867   +5.000000000000000 *
**************************************************
```

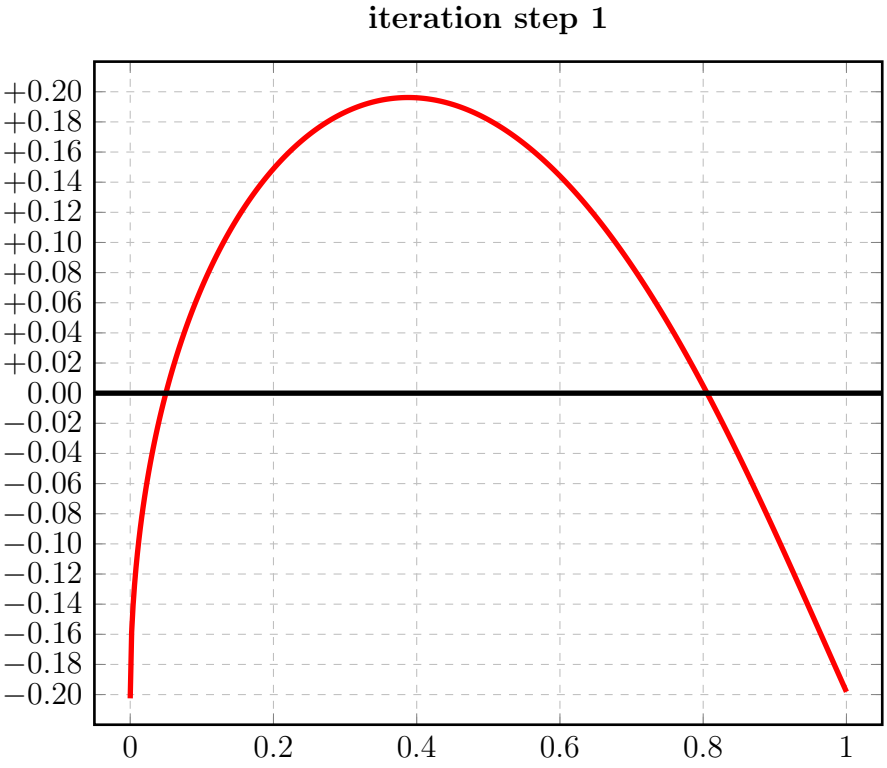
```
BRAESS - iteration step 1
Factor c=1.95e-03=2^-9 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.204932910211054   +1.982421235662245 *
* i= 2:   -0.002428031987120   +4.781856727993766 *
*                                                 *
*************************************************

-------
OMITTED: iteration steps 2-99
-------

BRAESS - iteration step 100
Factor c=7.63e-06=2^-17 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.206223833512365   +2.011214343485711 *
* i= 2:   -0.004035545235666   +4.430039331256711 *
*************************************************

-------
OMITTED: iteration steps 101 - 199
-------

BRAESS - iteration step 200
Factor c=7.63e-06=2^-17 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.206721084134290   +2.019546921769936 *
* i= 2:   -0.004612302376054   +4.344558621788245 *
*************************************************
```

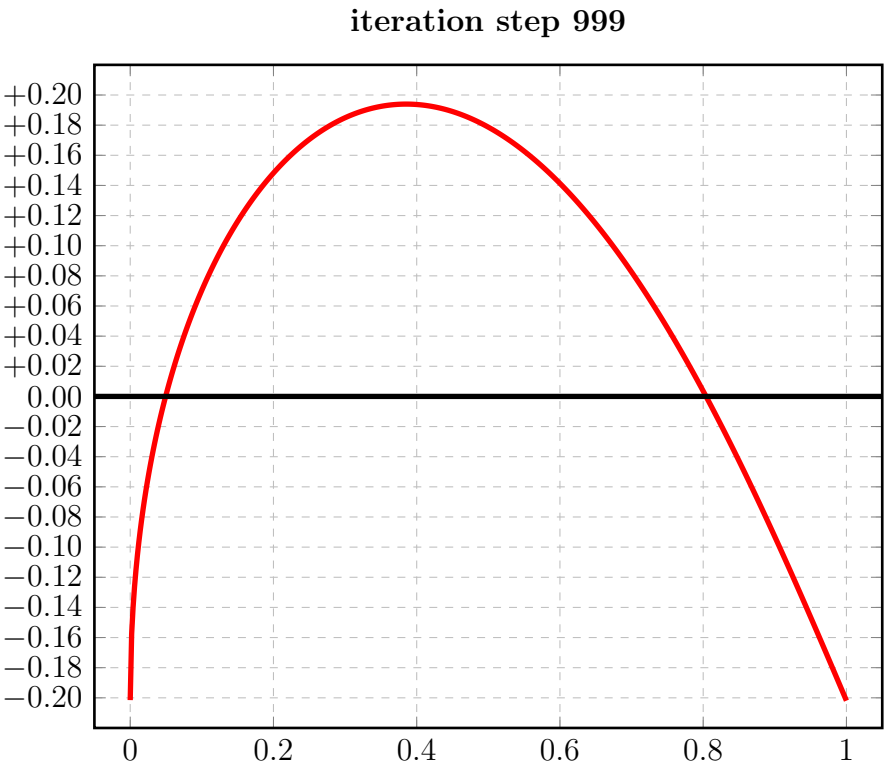
```
-------
OMITTED: iteration steps 201 - 299
-------

BRAESS - iteration step 300
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.207310801535274   +2.028430954825420 *
* i= 2:   -0.005282070063867   +4.259530979250022 *
**************************************************

-------
OMITTED: iteration steps 301 - 399
-------

BRAESS - iteration step 400
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208011487310966   +2.037927970325688 *
* i= 2:   -0.006063343807904   +4.174882350025880 *
**************************************************

-------
OMITTED: iteration steps 401 - 499
-------

BRAESS - iteration step 500
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.208846204796246   +2.048107930956127 *
* i= 2:   -0.006979177577540   +4.090550405820358 *
**************************************************
```

```
-------
OMITTED: iteration steps 501 - 599
-------

BRAESS - iteration step 600
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209845076505842   +2.059059922675827 *
* i= 2:   -0.008059666507886   +4.006401765169525 *
**************************************************

-------
OMITTED: iteration steps 601 - 699
-------

BRAESS - iteration step 700
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.211047104510063   +2.070889935131432 *
* i= 2:   -0.009343805006514   +3.922302037715874 *
**************************************************

-------
OMITTED: iteration steps 701 - 799
-------

BRAESS - iteration step 800
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.212503894594596   +2.083726996443030 *
* i= 2:   -0.010883198543557   +3.838098237594224 *
**************************************************
```

```
-------
OMITTED: iteration steps 801 - 899
-------

BRAESS - iteration step 900
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.213880009389713   +2.094683550731771 *
* i= 2:   -0.012324913970232   +3.771391686286841 *
**************************************************

-------
OMITTED: iteration steps 901 - 998
-------

BRAESS - iteration step 999
Factor c=3.81e-06=2^-18 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.214865375361337   +2.101962160155440 *
* i= 2:   -0.013351538127721   +3.729461687217995 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.22.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+05/t+05step0_x":
A local extremum has been found at x=  +0.389477096350347:
           x                  err(x)
  ------------------     ------------------
  +0.389477096350347     +0.196877255156456
  +0.000000000000000     -0.202698353304186 (*)
  +1.000000000000000     -0.187728254274083
A zero has been found at   x=  +0.049842159703097
A zero has been found at   x=  +0.809558782287683
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+05/t+05step1_x":
A local extremum has been found at x=  +0.388093080754274:
           x                  err(x)
  ------------------     ------------------
  +0.388093080754274     +0.196177165948342
  +0.000000000000000     -0.202504878223934
  +1.000000000000000     -0.198147918883857
A zero has been found at   x=  +0.049776309862166
A zero has been found at   x=  +0.805459987198816
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+05/t+05step999_x":
A local extremum has been found at x=  +0.388093080754274:
           x                  err(x)
  ------------------     ------------------
  +0.388093080754274     +0.196177165948342
  +0.000000000000000     -0.202504878223934 (*)
  +1.000000000000000     -0.198147918883857
A zero has been found at   x=  +0.049776309862166
A zero has been found at   x=  +0.805459987198816
```

## 3.23 Berechnung für $t_2 = +6.0$

### 3.23.1 Die Berechnung

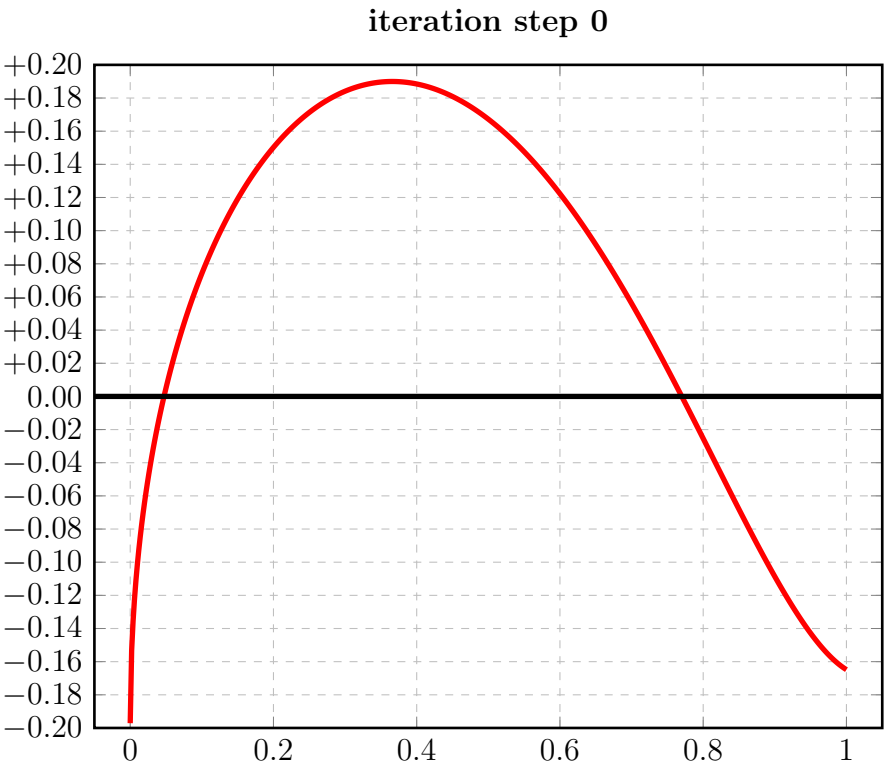
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+06-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +6.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=2.50e-01=2^-2 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.198166578192922   +2.080942592056198 *
* i= 2:   -0.001048223731207   +6.000000000000000 *
**************************************************
```

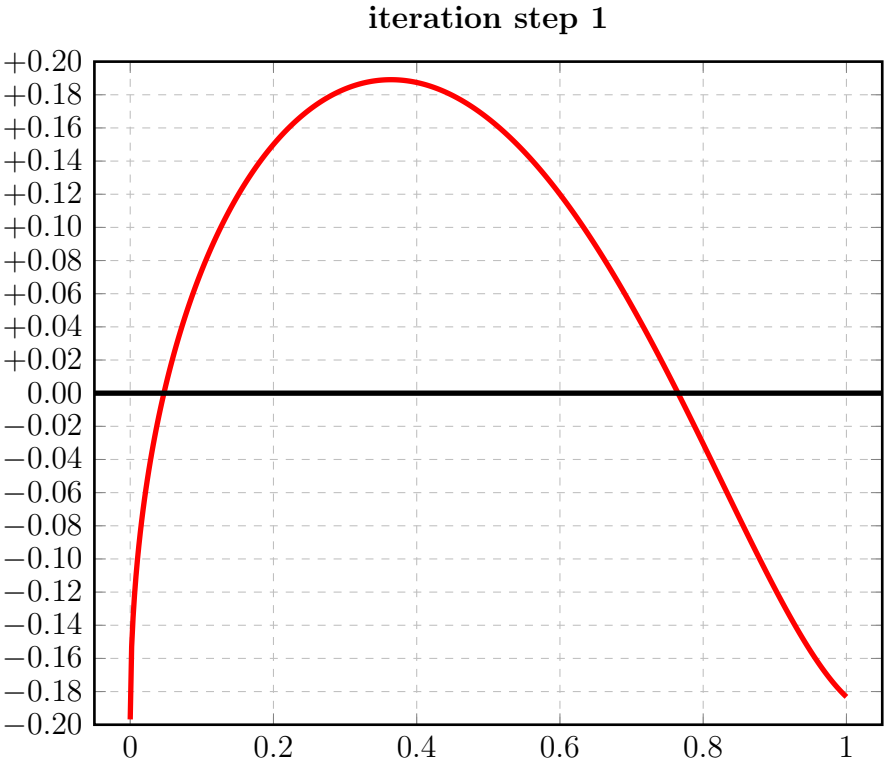
```
BRAESS - iteration step 1
Factor c=3.91e-03=2^-8 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.198188206123862   +2.098790428757948 *
* i= 2:   -0.001388959788899   +5.742815282479996 *
*************************************************

-------
OMITTED: iteration steps 2-99
-------

BRAESS - iteration step 100
Factor c=1.53e-05=2^-16 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.198745780731960   +2.139408100670371 *
* i= 2:   -0.002558779303172   +5.258841095095510 *
*************************************************

-------
OMITTED: iteration steps 101 - 199
-------

BRAESS - iteration step 200
Factor c=1.53e-05=2^-16 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.198998211903061   +2.149619794810940 *
* i= 2:   -0.002945081140008   +5.157267174127990 *
*************************************************
```

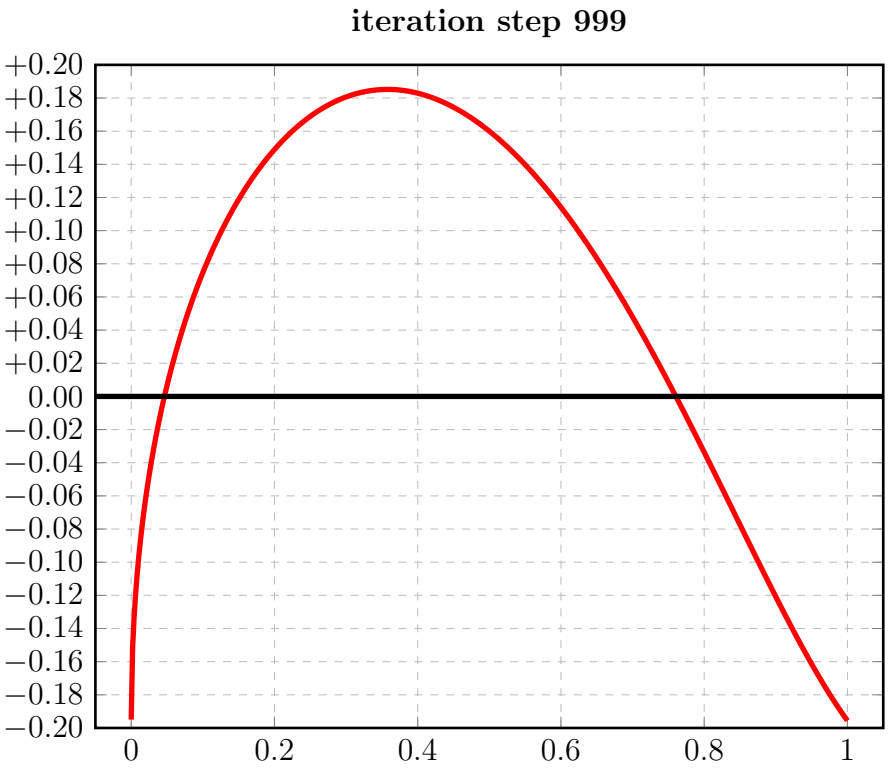
```
-------
OMITTED: iteration steps 201 - 299
-------

BRAESS - iteration step 300
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.199311674165329   +2.160431861989867 *
* i= 2:   -0.003393612394282   +5.056435040073699 *
**************************************************

-------
OMITTED: iteration steps 301 - 399
-------

BRAESS - iteration step 400
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.199698134599767   +2.171906041279253 *
* i= 2:   -0.003916228422356   +4.956226235137978 *
**************************************************

-------
OMITTED: iteration steps 401 - 499
-------

BRAESS - iteration step 500
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.200172325167551   +2.184115348087452 *
* i= 2:   -0.004527674781579   +4.856513633077051 *
**************************************************
```

```
-------
OMITTED: iteration steps 501 - 599
-------

BRAESS - iteration step 600
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.200752622985361   +2.197143135178576 *
* i= 2:   -0.005246332034617   +4.757180917156696 *
**************************************************

-------
OMITTED: iteration steps 601 - 699
-------

BRAESS - iteration step 700
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.201462211672592   +2.211084971349893 *
* i= 2:   -0.006095368671234   +4.658114864878510 *
**************************************************

-------
OMITTED: iteration steps 701 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.202330990524903   +2.226056713275280 *
* i= 2:   -0.007104680768948   +4.559172177938223 *
**************************************************
```

```
-------
OMITTED: iteration steps 801 - 899
-------

BRAESS - iteration step 900
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.203398125975962   +2.242199162105570 *
* i= 2:   -0.008313437098042   +4.460180889802309 *
**************************************************

-------
OMITTED: iteration steps 901 - 998
-------

BRAESS - iteration step 999
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.204701142396667   +2.259501989142757 *
* i= 2:   -0.009757748291970   +4.361937293149654 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.20
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

### 3.23.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+06/t+06step0_x":
A local extremum has been found at x=  +0.365926678806186:
           x                  err(x)
  ------------------     ------------------
  +0.365926678806186     +0.189977807529621
  +0.000000000000000     -0.197118354461715 (*)
  +1.000000000000000     -0.164830441378146
A zero has been found at   x=  +0.047184673031470
A zero has been found at   x=  +0.770088088387613
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+06/t+06step1_x":
A local extremum has been found at x=  +0.364113463495686:
           x                  err(x)
  ------------------     ------------------
  +0.364113463495686     +0.189095577820639
  +0.000000000000000     -0.196799246334963
  +1.000000000000000     -0.183208100602218
A zero has been found at   x=  +0.047064995144342
A zero has been found at   x=  +0.764875901932200
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+06/t+06step999_x":
A local extremum has been found at x=  +0.357984009805017:
           x                  err(x)
  ------------------     ------------------
  +0.357984009805017     +0.185195284373666
  +0.000000000000000     -0.194943394104697
  +1.000000000000000     -0.195598414052391 (*)
A zero has been found at   x=  +0.046386160334877
A zero has been found at   x=  +0.760443204856692
```

## 3.24 Berechnung für $t_2 = +7.0$

### 3.24.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+07-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +7.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.187716633946043   +2.285158219629735 *
* i= 2:   -0.000642589213392   +7.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=3.91e-03=2^-8 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.187602840014212   +2.296630824970095 *
* i= 2:   -0.000760499985830   +6.845021194429231 *
*                                                 *
*************************************************

-------
OMITTED: iteration steps 2-179
-------

BRAESS - iteration step 180
Factor c=1.22e-04=2^-13 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.191386132795072   +2.516608127341494 *
* i= 2:   -0.007583362670241   +5.064432446587126 *
*************************************************
```

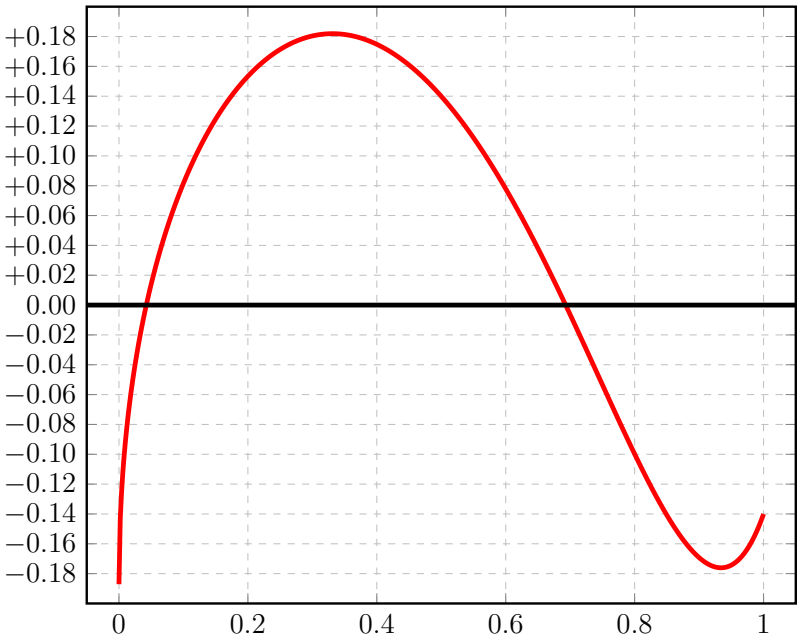
(*Vgl. [1] §7: Hinweis zu den Berechnungen von Abschnitt 7.3 mit "Startfunktionen mit* $t_2 \geq 2.0$*" und "Zeichnung 26".*)

```
-------
OMITTED: iteration steps 181 - 399
-------

BRAESS - iteration step 400
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.203276529412459   +2.668940958966348 *
* i= 2:   -0.020616405437149   +4.446719194947629 *
*************************************************
```

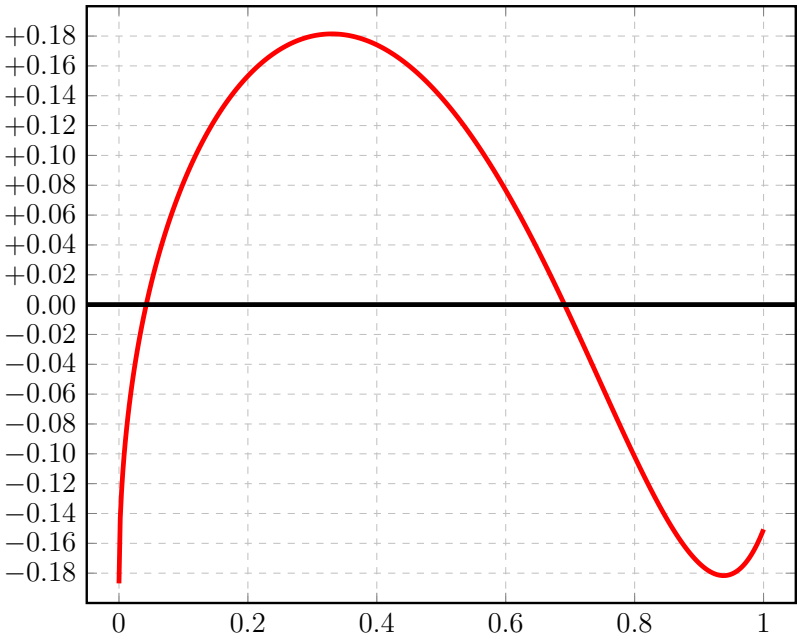
```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.215621911627427   +2.754995861128058 *
* i= 2:   -0.033399902296711   +4.195098822340406 *
**************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.227912601810306   +2.814820005556556 *
* i= 2:   -0.045929281095052   +4.047717833008846 *
**************************************************

-------
OMITTED: iteration steps 801 - 998
-------

BRAESS - iteration step 999
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.238943288794652   +2.856369248335398 *
* i= 2:   -0.057098947546757   +3.956010021732389 *
**************************************************
```

```
End of Braess iteration - limit for number of iteration steps reached
```

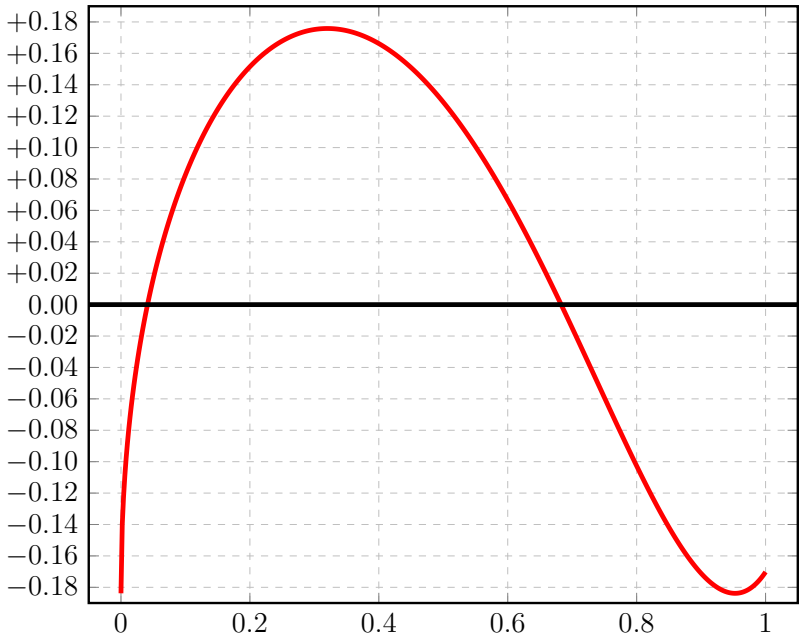
**iteration step 0**




**iteration step 1**

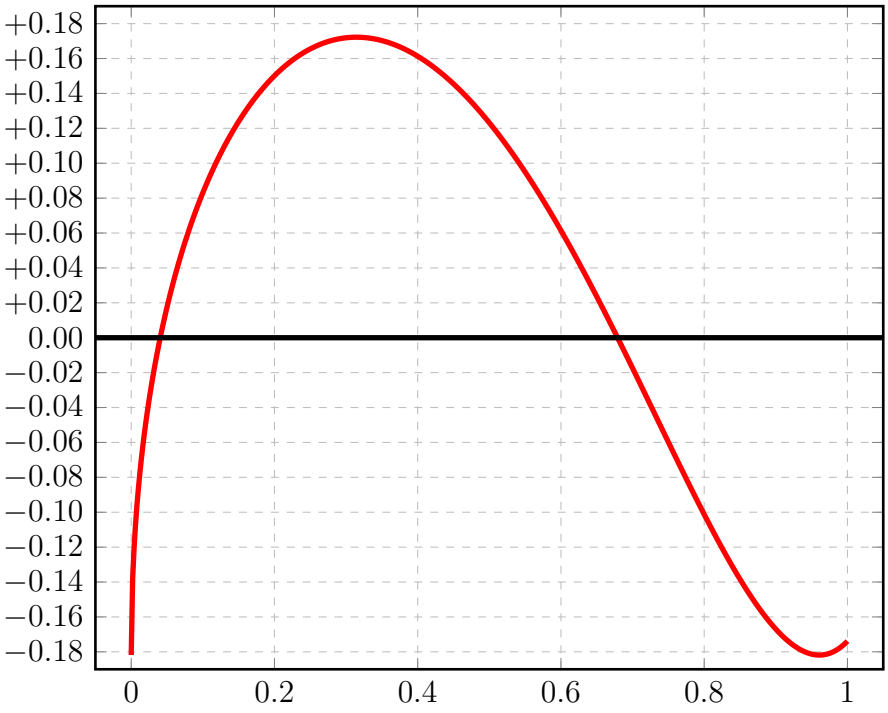


**iteration step 180**



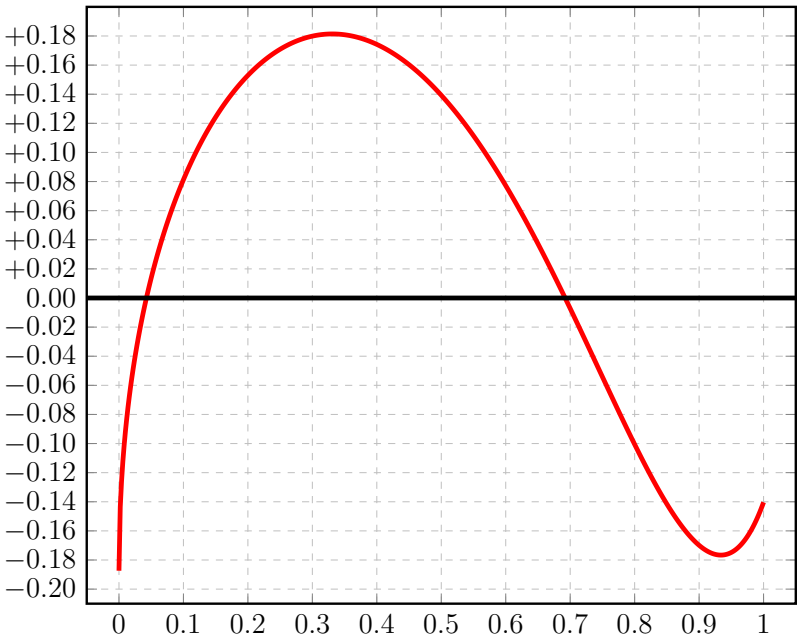

(*Vgl. "Zeichnung 26" aus [1] §7 unten!*)

**iteration step 999**

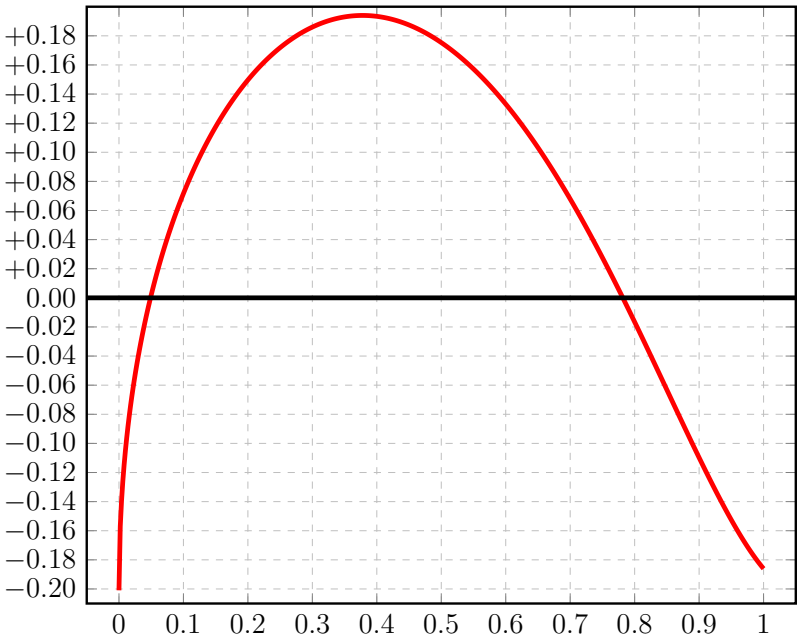


*Zum Vergleich: die Startfunktionen von [1] nach Fall 2 (“Zeichnung 14D”) und Fall 1 (“Zeichnung 14I”):*

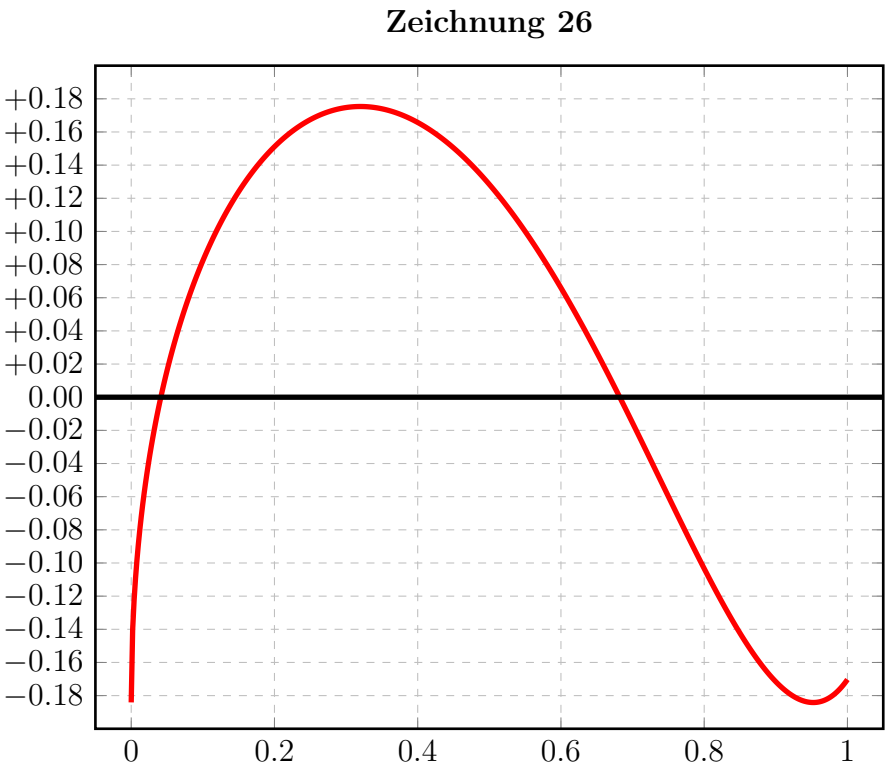
**Zeichnung 14D von [1], §7.2**




**Zeichnung 14I von [1], §7.2**



*Zum Vergleich mit iteration step 180: "Zeichnung 26" aus [1]:*



### 3.24.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+07/t+07step0_x":
A local extremum has been found at x=  +0.331251449361783:
           x                  err(x)
  ------------------     ------------------
  +0.331251449361783     +0.181902726194936
  +0.000000000000000     -0.187074044732651 (*)
  +1.000000000000000     -0.140051955980535
A zero has been found at   x=  +0.042419210887465
A zero has been found at   x=  +0.692974609392343
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+07/t+07step1_x":
A local extremum has been found at x=  +0.160835014031401:
           x                  err(x)
  ------------------     ------------------
  +0.330283326065617     +0.181435569518976
  +0.000000000000000     -0.186842340028382
  +1.000000000000000     -0.150634809030498
A zero has been found at   x=  +0.042329089269584
A zero has been found at   x=  +0.691264234238101
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+07/t+07step999_x":
A local extremum has been found at x=  +0.314340319510541:
           x                  err(x)
  ------------------     ------------------
  +0.314340319510541     +0.172214566015249
  +0.000000000000000     -0.181844341247895 (*)
  +1.000000000000000     -0.173862603835352
A zero has been found at   x=  +0.040490057036557
A zero has been found at   x=  +0.679067549595442
```

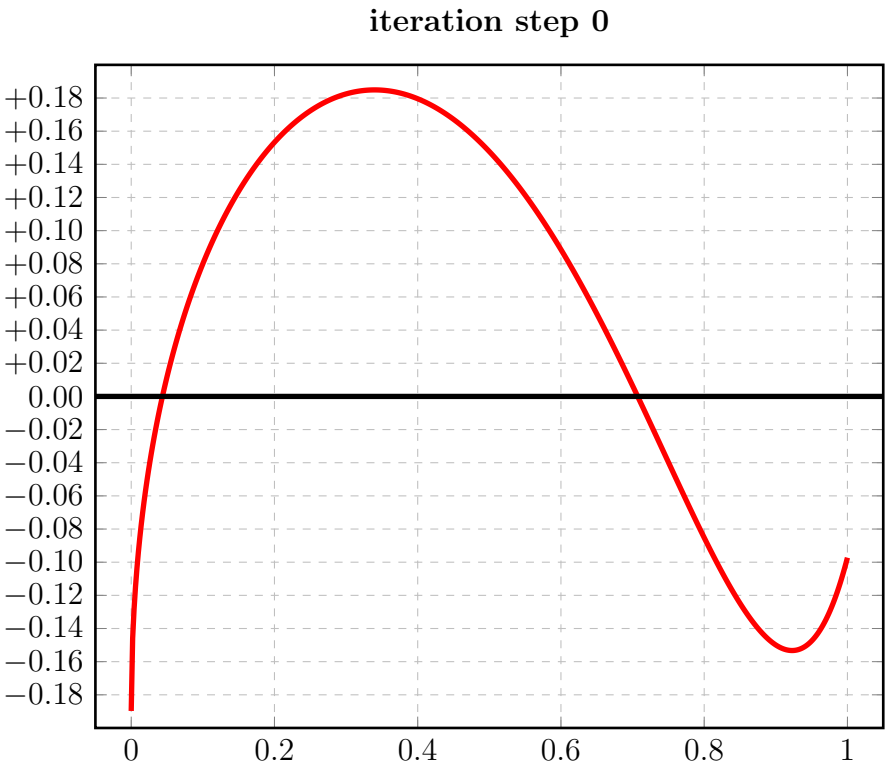
## 3.25 Berechnung für $t_2 = +8.0$

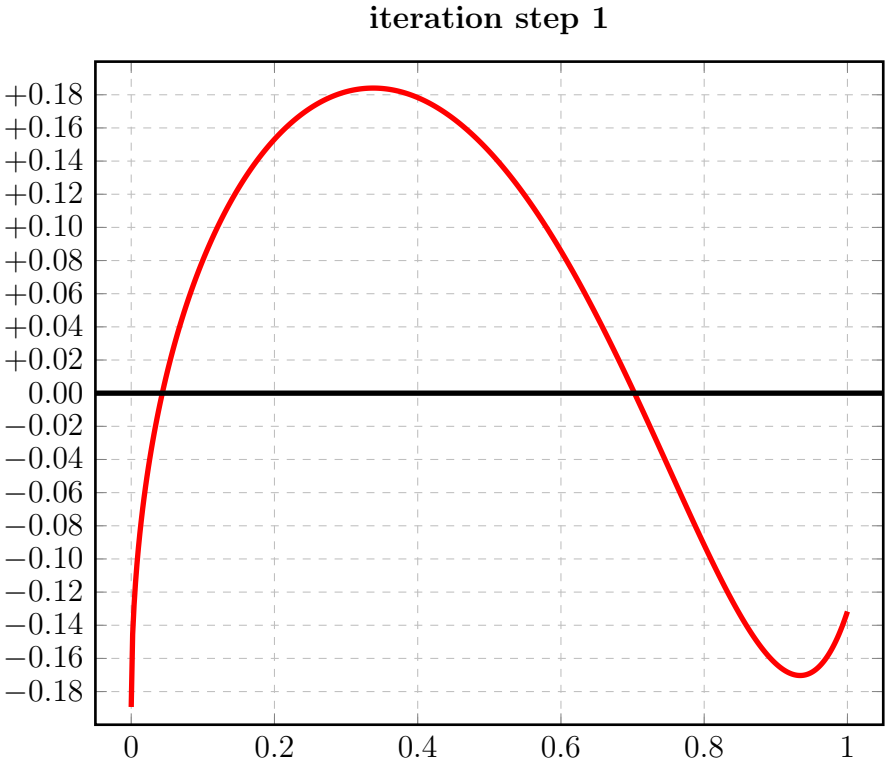
### 3.25.1 Die Berechnung

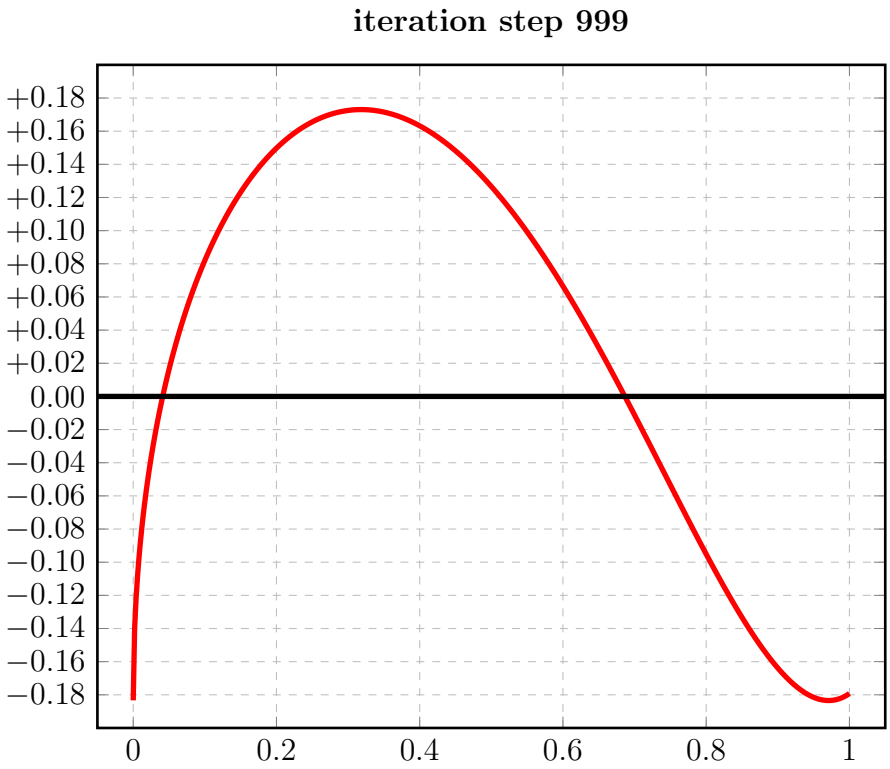
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+08-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +8.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.189937824855173   +2.200349676547716 *
* i= 2:   -0.000207157410086   +8.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=7.81e-03=2^-7 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.189585191515736   +2.217355570790526 *
* i= 2:   -0.000283697515104   +7.672114792246329 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=1.22e-04=2^-13 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.190916649108410   +2.444896934492402 *
* i= 2:   -0.005367919840449   +5.251940043520245 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.199382649090007   +2.585965106242461 *
* i= 2:   -0.015065292034476   +4.579710165169716 *
*************************************************
```

```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.207936233466742   +2.663334443735901 *
* i= 2:   -0.024089724608080   +4.315921366975935 *
*************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.217299163642238   +2.723177178576383 *
* i= 2:   -0.033742755084870   +4.145190451178070 *
*************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=7.63e-06=2^-17 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.227988816262704   +2.774831347956592 *
* i= 2:   -0.044639590060983   +4.016101158909810 *
*************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.25.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+08/t+08step0_x":
A local extremum has been found at x=  +0.339852278605565:
           x                  err(x)
  ------------------     ------------------
  +0.339852278605565     +0.184897240406470
  +0.000000000000000     -0.189730667445087 (*)
  +1.000000000000000     -0.097263414555366
A zero has been found at   x=  +0.043580640026708
A zero has been found at   x=  +0.707138329479409
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+08/t+08step1_x":
A local extremum has been found at x=  +0.337889658197099:
           x                  err(x)
  ------------------     ------------------
  +0.337889658197099     +0.184033678307182
  +0.000000000000000     -0.189301494000632
  +1.000000000000000     -0.131689630660551
A zero has been found at   x=  +0.043407134327485
A zero has been found at   x=  +0.702713188260905
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+08/t+08step999_x":
A local extremum has been found at x=  +0.318239544314866:
           x                  err(x)
  ------------------     ------------------
  +0.318239544314866     +0.173028865376774
  +0.000000000000000     -0.183349226201721 (*)
  +1.000000000000000     -0.179211902655792
A zero has been found at   x=  +0.041175407752790
A zero has been found at   x=  +0.686434923245337
```

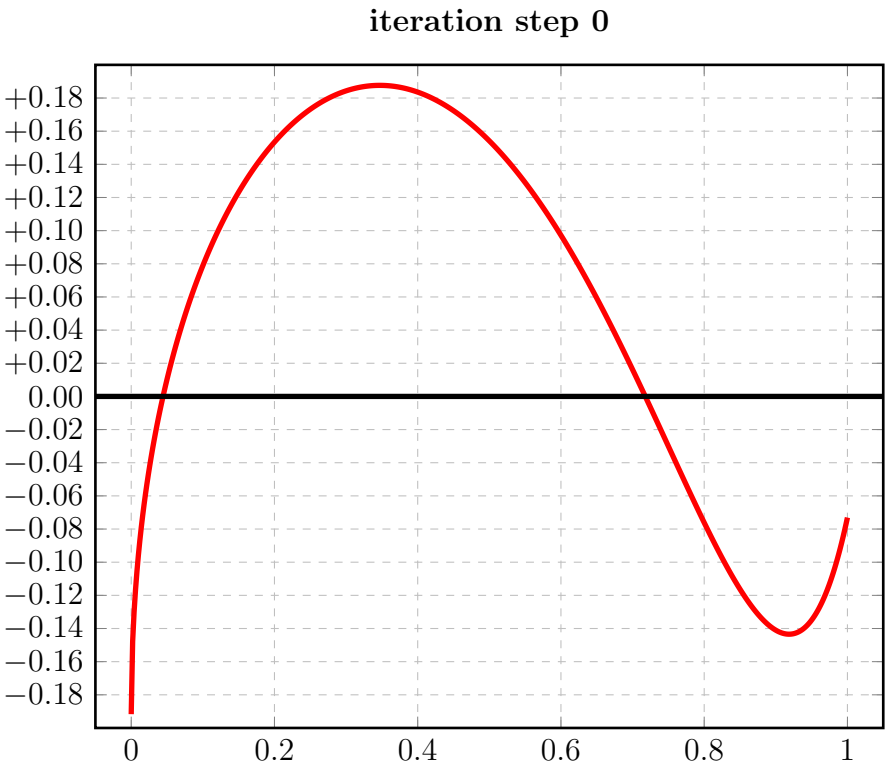
## 3.26 Berechnung für $t_2 = +9.0$

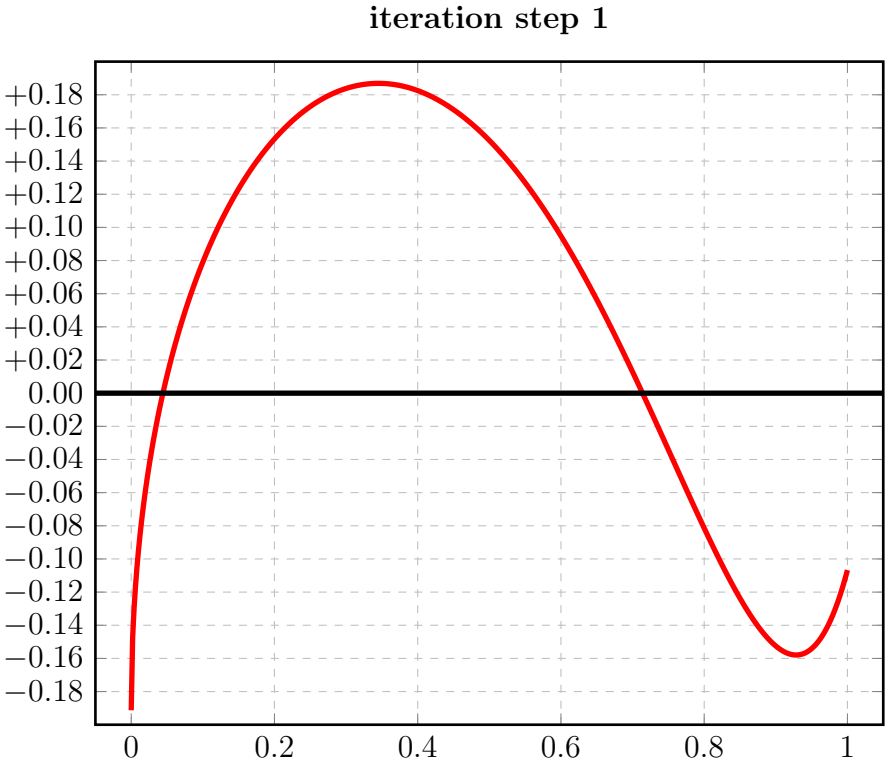
### 3.26.1 Die Berechnung

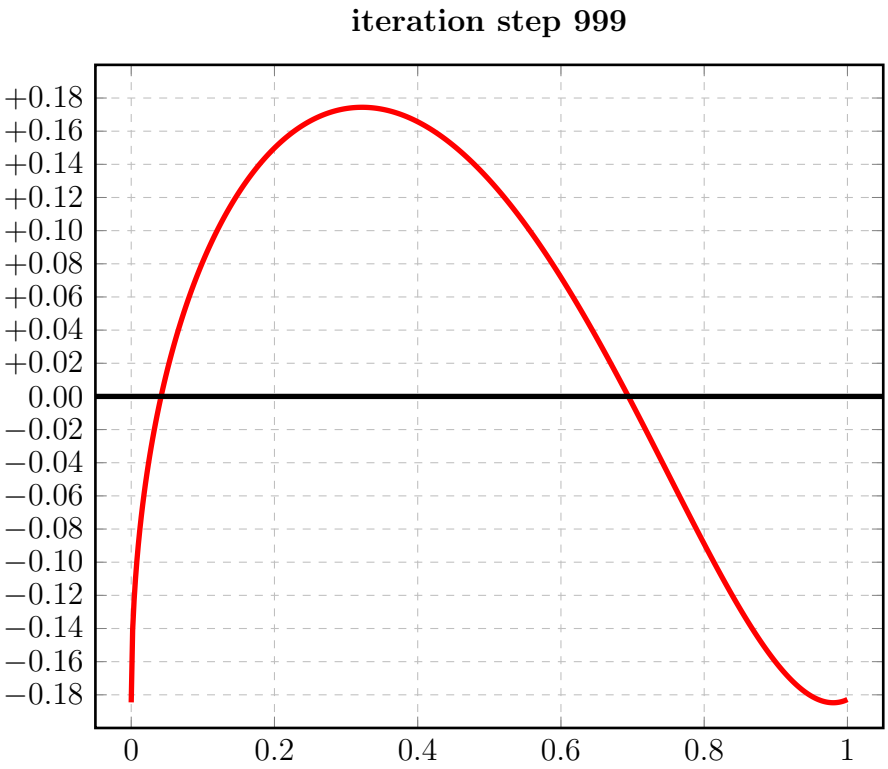
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+09-terse":
----------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                 a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000   +9.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.191744769295847   +2.140516498029099 *
* i= 2:   -0.000068799836357   +9.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=7.81e-03=2^-7 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.191377508466139   +2.153937183121003 *
* i= 2:   -0.000095344062920   +8.646877256097383 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=1.22e-04=2^-13 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.190471536261575   +2.373120973476249 *
* i= 2:   -0.003446215475015   +5.523530327693515 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.195483616326919   +2.495407860097245 *
* i= 2:   -0.009734289193378   +4.804716683299619 *
*************************************************
```

```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=3.05e-05=2^-15 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.201854629503275   +2.574807143950503 *
* i= 2:   -0.016695077620838   +4.476243313357449 *
**************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.209058666450316   +2.636034944958379 *
* i= 2:   -0.024258759197592   +4.270129531544898 *
**************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.216859582458107   +2.685915262338356 *
* i= 2:   -0.032302575940727   +4.125177065136464 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.26.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+09/t+09step0_x":
A local extremum has been found at x=  +0.347083894947787:
           x                  err(x)
  ------------------     ------------------
  +0.347083894947787     +0.187634724698924
  +0.000000000000000     -0.191675969459490 (*)
  +1.000000000000000     -0.073073825075451
A zero has been found at   x=  +0.044424124998328
A zero has been found at   x=  +0.717750379217902
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+09/t+09step1_x":
A local extremum has been found at x=  +0.345295095762686:
           x                  err(x)
  ------------------     ------------------
  +0.345295095762686     +0.186886563189865
  +0.000000000000000     -0.191282164403219
  +1.000000000000000     -0.106699021501036
A zero has been found at   x=  +0.044259850326996
A zero has been found at   x=  +0.713913360871945
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+09/t+09step999_x":
A local extremum has been found at x=  +0.322314622464676:
           x                  err(x)
  ------------------     ------------------
  +0.322314622464676     +0.174408271436307
  +0.000000000000000     -0.184557006517380 (*)
  +1.000000000000000     -0.182838671764896
A zero has been found at   x=  +0.041694519631054
A zero has been found at   x=  +0.694090209981158
```

## 3.27 Berechnung für $t_2 = +10.0$

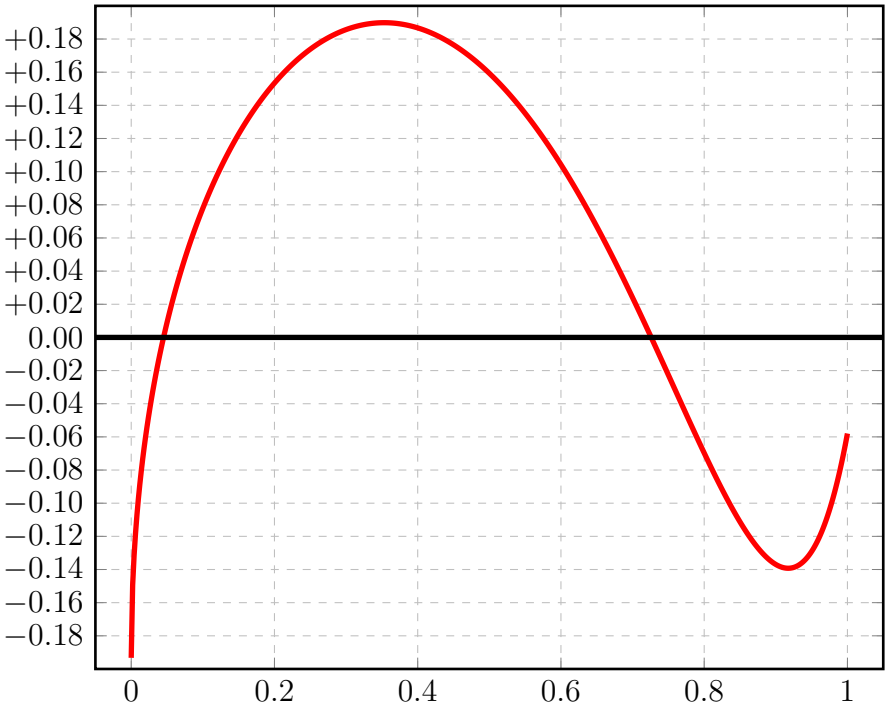
### 3.27.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+10-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  +10.000000000000000

------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         -----------------   ----------------- *
* i= 1:   +0.193309205305721   +2.095772444904453 *
* i= 2:   -0.000023334579988  +10.000000000000000 *
**************************************************
```

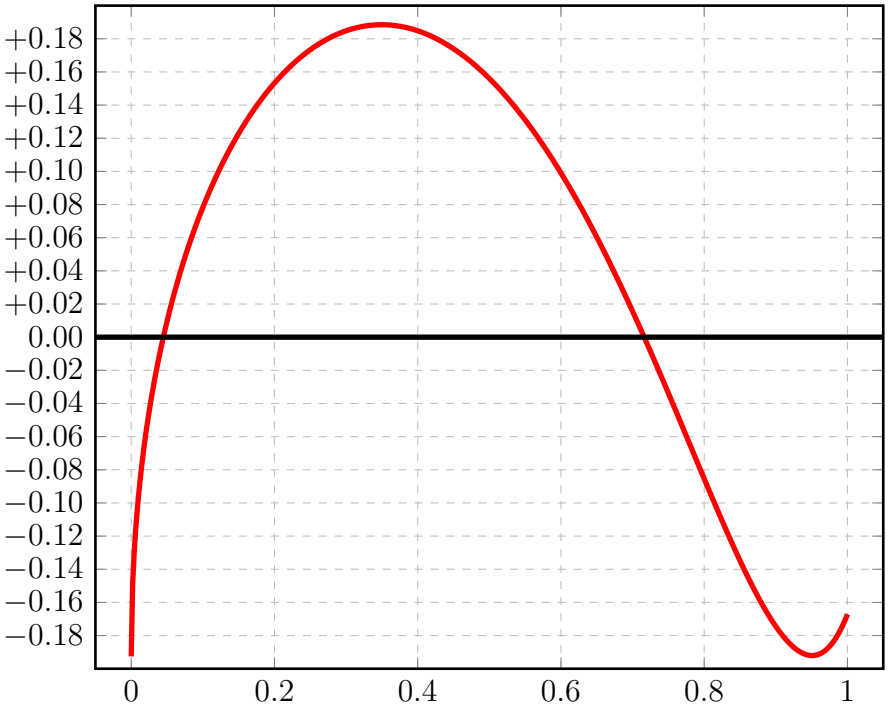
```
BRAESS - iteration step 1
Factor c=1.56e-02=2^-6 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192604085634037   +2.117950990603124 *
* i= 2:   -0.000042542009014   +9.230838144856465 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=1.22e-04=2^-13 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.190391572533707   +2.273960897719996 *
* i= 2:   -0.001390562502334   +6.172898228850281 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192147790408764   +2.380879953174006 *
* i= 2:   -0.004630577903202   +5.261935552426610 *
*************************************************
```

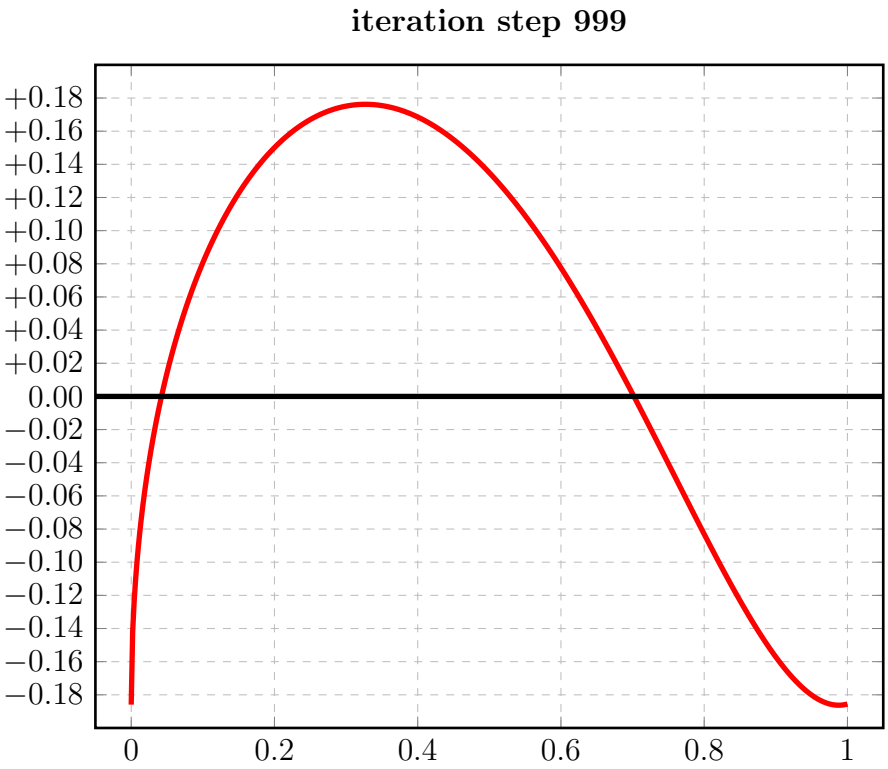
```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=3.05e-05=2^-15 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.196822956076777   +2.477270954372712 *
* i= 2:   -0.010241733604368   +4.728301338771104 *
**************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.201323167553498   +2.533611391617675 *
* i= 2:   -0.015162820872575   +4.489658138747622 *
**************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=1.53e-05=2^-16 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.205324972754462   +2.571982855927718 *
* i= 2:   -0.019408559717403   +4.349188304884487 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```

**iteration step 0**




**iteration step 1**




iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.27.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+10/t+10step0_x":
A local extremum has been found at x=  +0.353190389557169:
           x                  err(x)
  ------------------     ------------------
  +0.353190389557169     +0.189848639667972
  +0.000000000000000     -0.193285870725733 (*)
  +1.000000000000000     -0.057957974111353
A zero has been found at   x=  +0.045135510248913
A zero has been found at   x=  +0.726039339245954
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+10/t+10step1_x":
A local extremum has been found at x=  +0.349660406699013:
           x                  err(x)
  ------------------     ------------------
  +0.349660406699013     +0.188480505599207
  +0.000000000000000     -0.192561543625023
  +1.000000000000000     -0.167096380694935
A zero has been found at   x=  +0.044826034101633
A zero has been found at   x=  +0.716226028635774
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+10/t+10step999_x":
A local extremum has been found at x=  +0.326942483821526:
           x                  err(x)
  ------------------     ------------------
  +0.326942483821526     +0.176208184363790
  +0.000000000000000     -0.185916413037059 (*)
  +1.000000000000000     -0.185539524771970
A zero has been found at   x=  +0.042261614934358
A zero has been found at   x=  +0.701907466233323
```

## 3.28 Berechnung für $t_2 = +11.0$

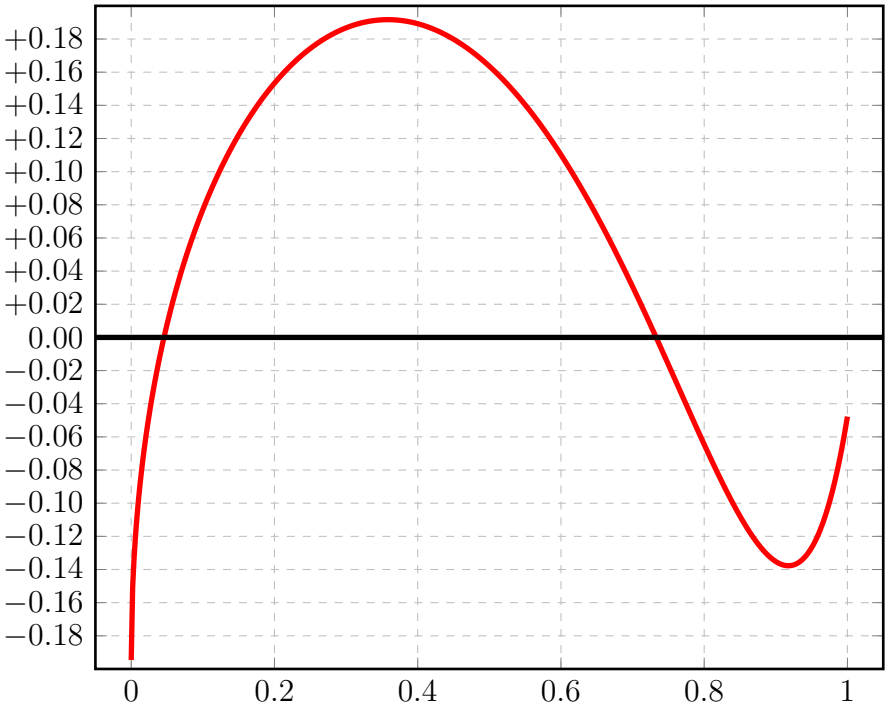
### 3.28.1 Die Berechnung

```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+11-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  +11.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.194633935300639   +2.061047763948363 *
* i= 2:   -0.000008032474105  +11.000000000000000 *
**************************************************
```

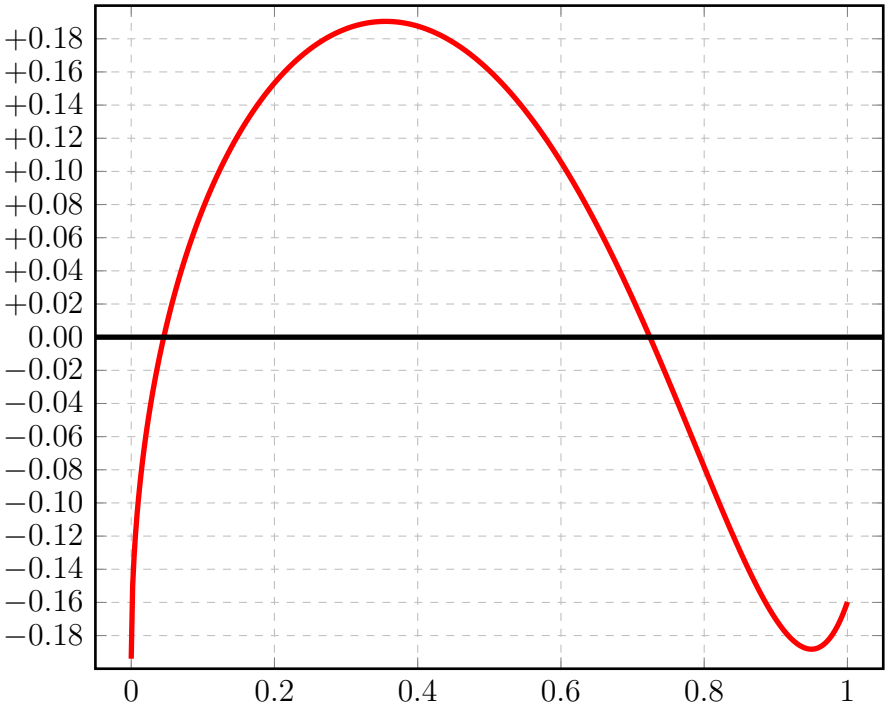
```
BRAESS - iteration step 1
Factor c=1.56e-02=2^-6 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193975023971821   +2.079880198427372 *
* i= 2:   -0.000015092554858  +10.166770376000779 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=2.44e-04=2^-12 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.191156882271551   +2.224177970789227 *
* i= 2:   -0.000860147533561   +6.515407765993439 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192110008726000   +2.331568137326579 *
* i= 2:   -0.003473838118187   +5.426128364865993 *
*************************************************
```

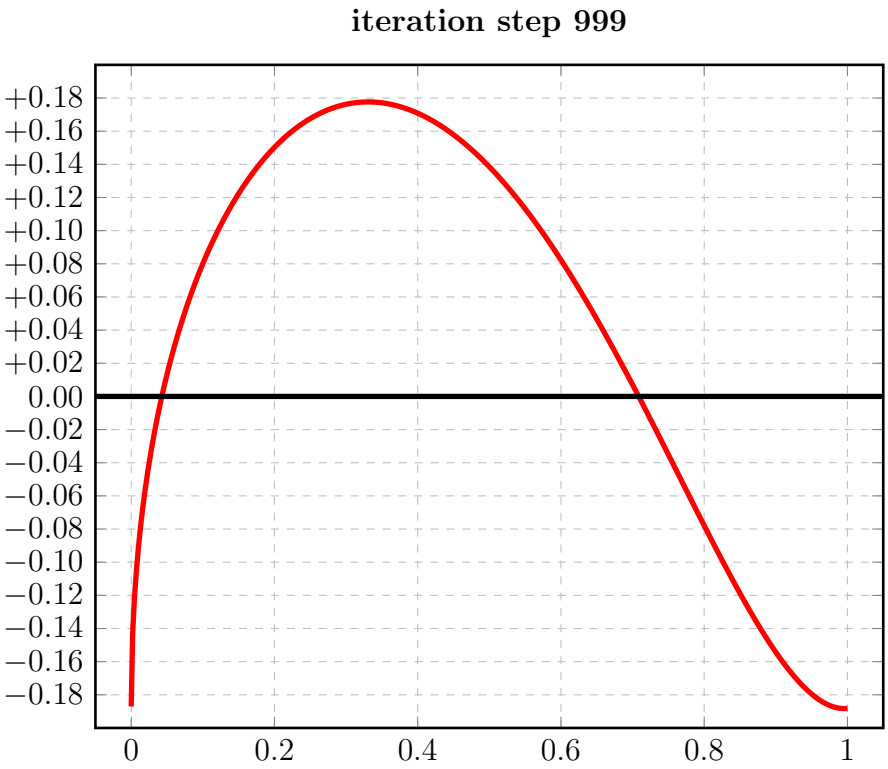
```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=3.05e-05=2^-15 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.194369257912314   +2.397547548209066 *
* i= 2:   -0.006473165109008   +4.987847057682122 *
**************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=3.05e-05=2^-15 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.197327387615397   +2.450641586672358 *
* i= 2:   -0.009913962078364   +4.709119540105748 *
**************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=7.63e-06=2^-17 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.201242341441152   +2.500792161444208 *
* i= 2:   -0.014210208695280   +4.489174686389523 *
**************************************************

End of Braess iteration - limit for number of iteration steps reached
```

**iteration step 0**




**iteration step 1**




iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.28.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+11/t+11step0_x":
A local extremum has been found at x=  +0.358424886713362:
           x                  err(x)
  ------------------     ------------------
  +0.358424886713362     +0.191677338377732
  +0.000000000000000     -0.194625902826534 (*)
  +1.000000000000000     -0.047755357989507
A zero has been found at   x=  +0.045736225245490
A zero has been found at   x=  +0.732938048067131
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+11/t+11step1_x":
A local extremum has been found at x=  +0.355223084016019:
           x                  err(x)
  ------------------     ------------------
  +0.355223084016019     +0.190484399633655
  +0.000000000000000     -0.193959931416963
  +1.000000000000000     -0.159714008361134
A zero has been found at   x=  +0.045445913708077
A zero has been found at   x=  +0.724202953960591
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+11/t+11step999_x":
A local extremum has been found at x=  +0.330796067639441:
           x                  err(x)
  ------------------     ------------------
  +0.330796067639441     +0.177639964085375
  +0.000000000000000     -0.187032132745872
  +1.000000000000000     -0.188186897441108 (*)
A zero has been found at   x=  +0.042735418173119
A zero has been found at   x=  +0.708531294513176
```

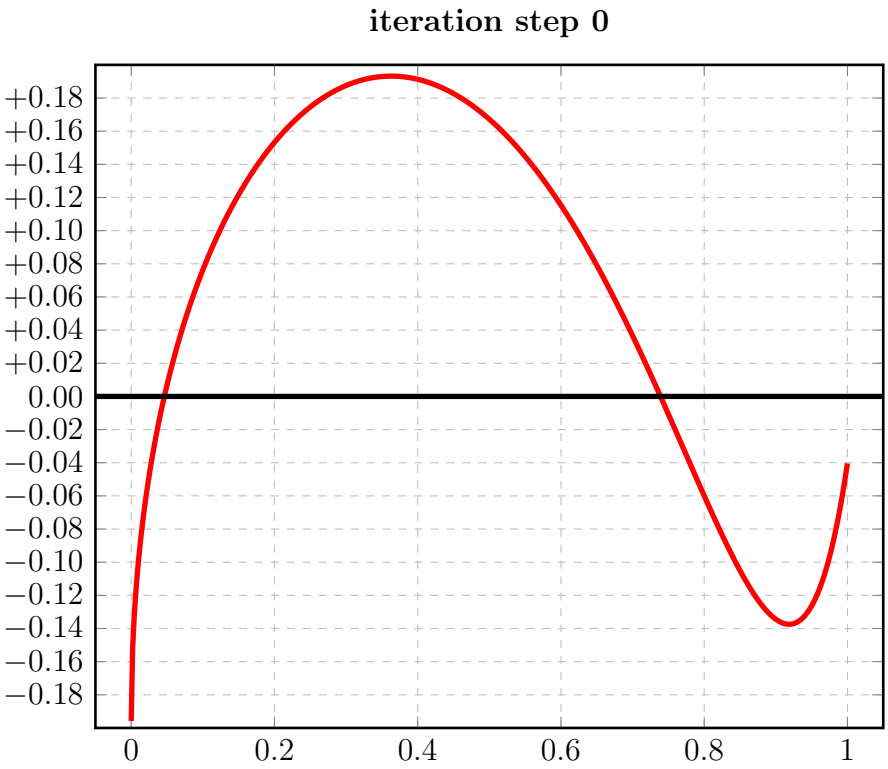
## 3.29 Berechnung für $t_2 = +12.0$

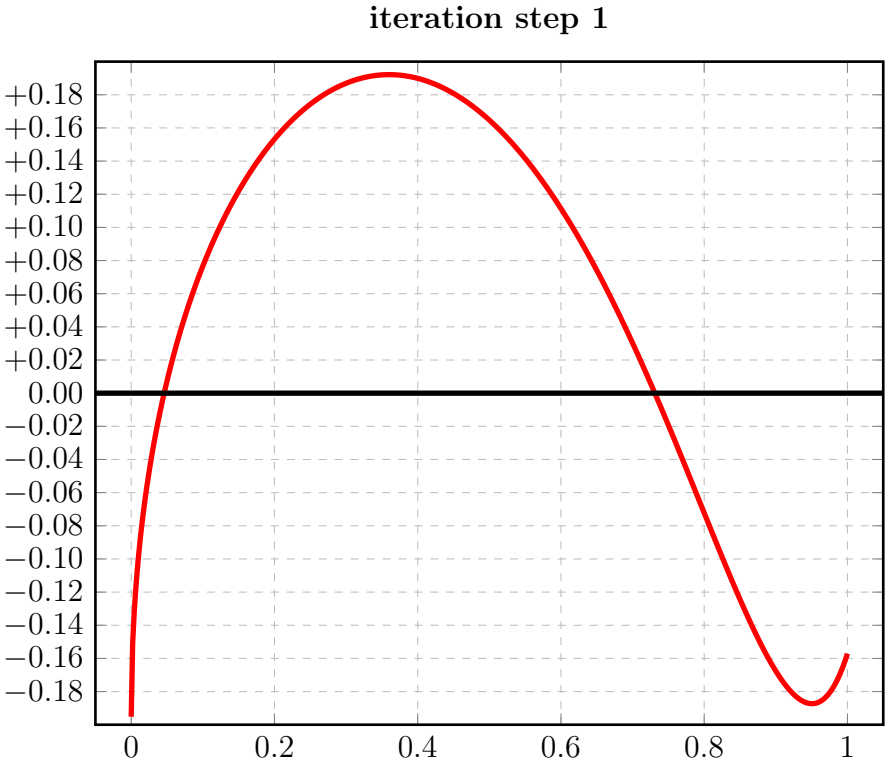
### 3.29.1 Die Berechnung

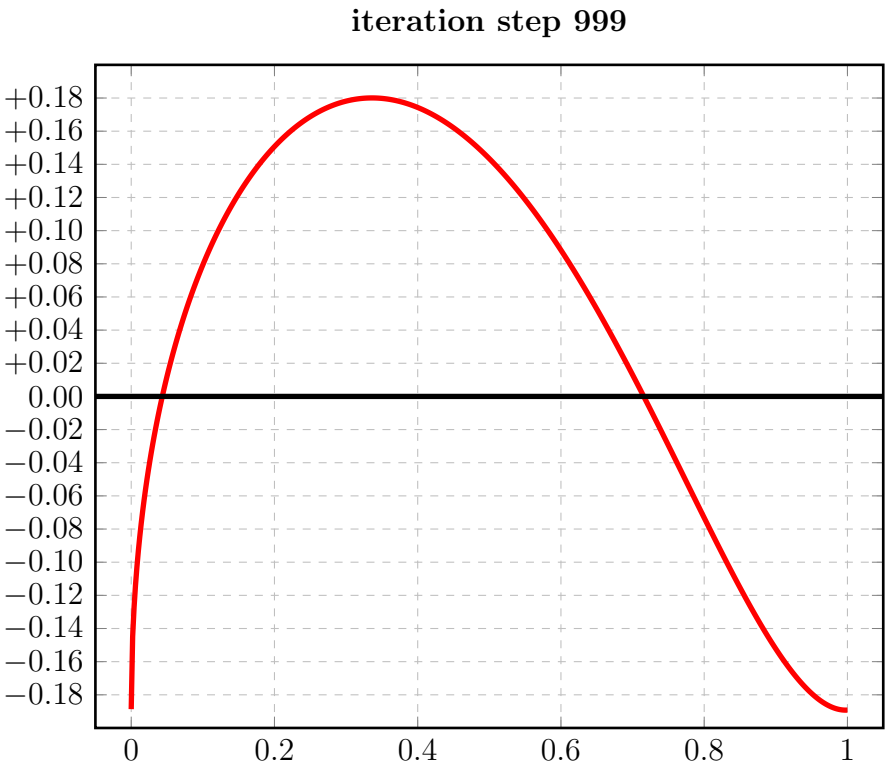
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+12-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                       : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  +12.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.195783028056321   +2.032933202797683 *
* i= 2:   -0.000002793753092  +12.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=1.56e-02=2^-6 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.195174604935244   +2.049300495675010 *
* i= 2:   -0.000005429420002  +11.096188809212554 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=2.44e-04=2^-12 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192248713677537   +2.164836650749223 *
* i= 2:   -0.000380281950125   +7.155651801021282 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.191877736831998   +2.247051921971959 *
* i= 2:   -0.001570582656778   +5.987664083275897 *
*************************************************
```

```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192800725491976   +2.310814066541507 *
* i= 2:   -0.003403919001981   +5.401229570321611 *
*************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=1.53e-05=2^-16 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.194145723342215   +2.354438748429816 *
* i= 2:   -0.005258310632195   +5.091782958267194 *
*************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=1.53e-05=2^-16 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.195131212823847   +2.377712484460028 *
* i= 2:   -0.006484161240392   +4.948842208857312 *
*************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.29.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+12/t+12step0_x":
A local extremum has been found at x=  +0.362996038319592:
           x                  err(x)
  ------------------     ------------------
  +0.362996038319592     +0.193202376710098
  +0.000000000000000     -0.195780234303229 (*)
  +1.000000000000000     -0.040391151971062
A zero has been found at   x=  +0.046261303571879
A zero has been found at   x=  +0.738997442882198
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+12/t+12step1_x":
A local extremum has been found at x=  +0.360062198272958:
           x                  err(x)
  ------------------     ------------------
  +0.360062198272958     +0.192141591468978
  +0.000000000000000     -0.195169175515242
  +1.000000000000000     -0.157132507906029
A zero has been found at   x=  +0.045991144112864
A zero has been found at   x=  +0.731077707739427
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+12/t+12step999_x":
A local extremum has been found at x=  +0.336128292332844:
           x                  err(x)
  ------------------     ------------------
  +0.336128292332844     +0.180053302335335
  +0.000000000000000     -0.188647051583455
  +1.000000000000000     -0.189214375479192 (*)
A zero has been found at   x=  +0.043388887957843
A zero has been found at   x=  +0.715599841547228
```

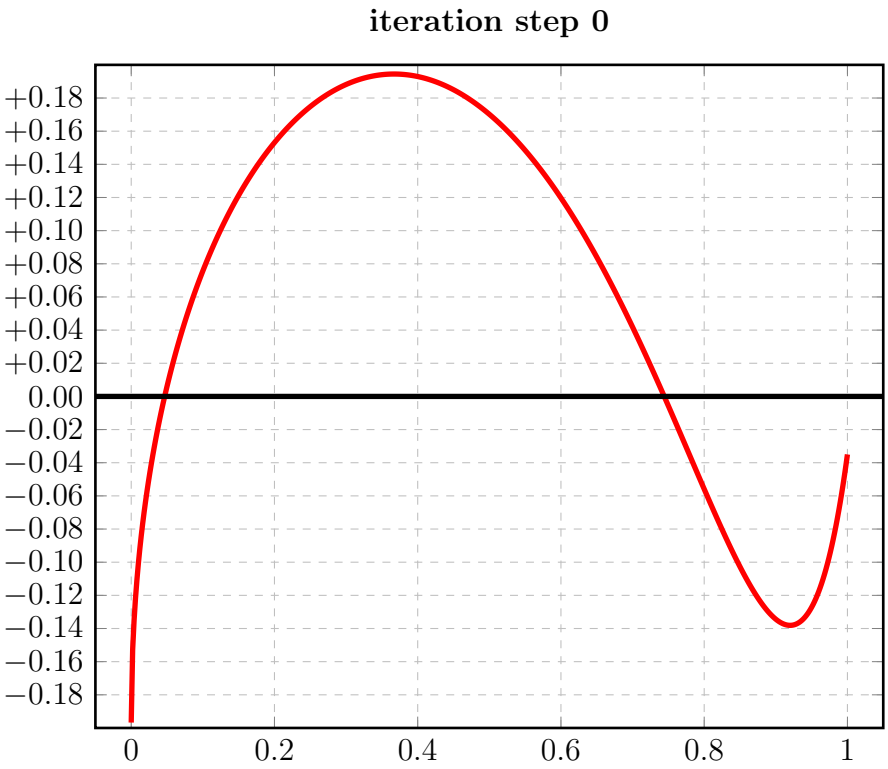
## 3.30 Berechnung für $t_2 = +13.0$

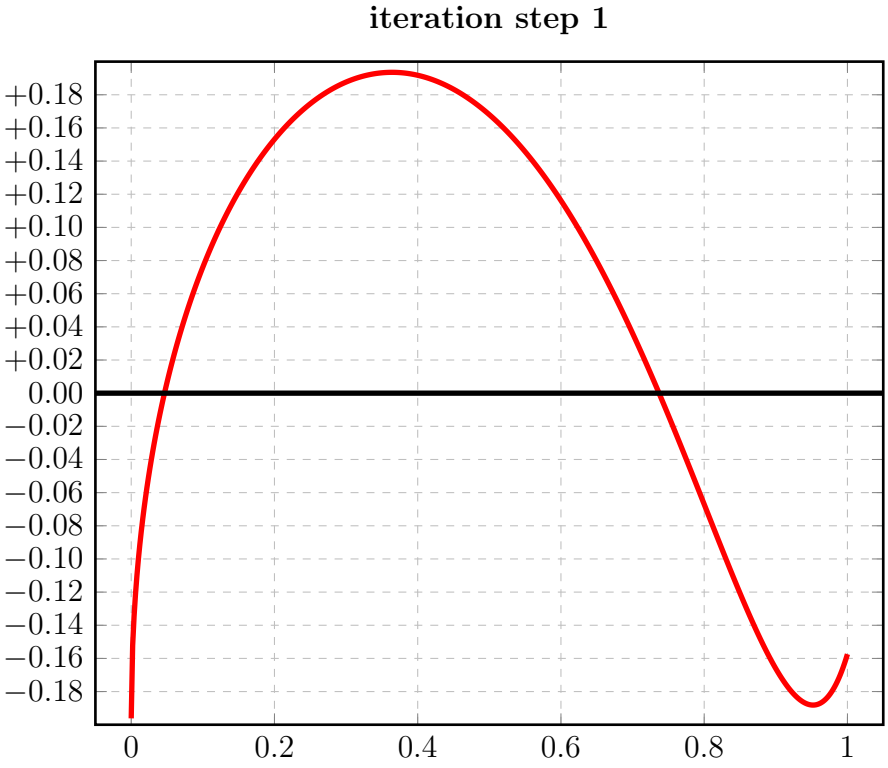
### 3.30.1 Die Berechnung

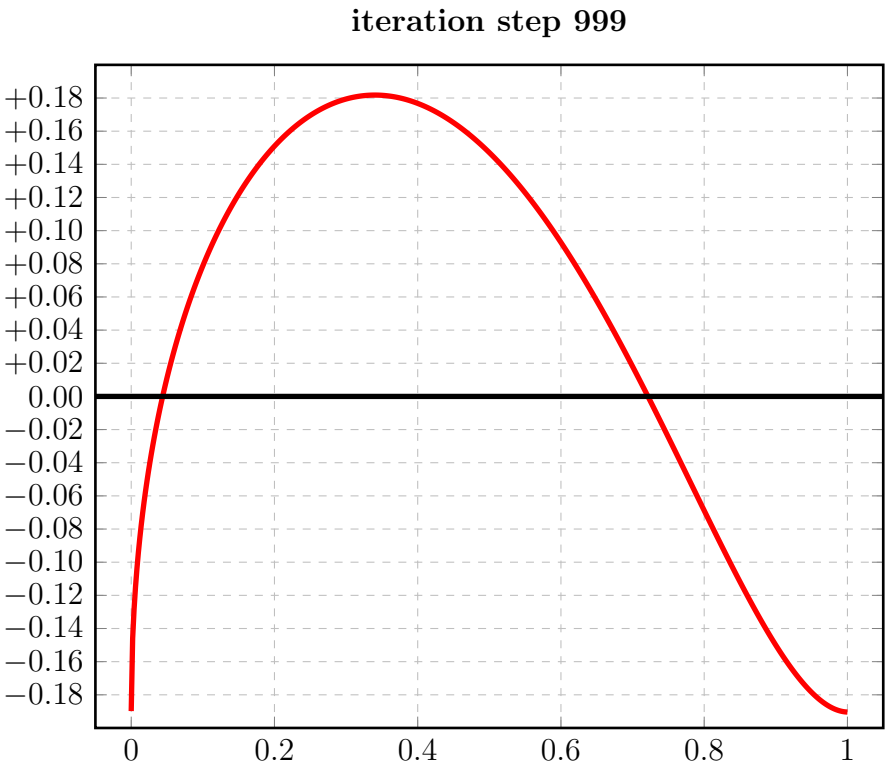
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+13-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  +13.000000000000000
-----------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.196743302404103   +2.010296650228070 *
* i= 2:   -0.000000980539140  +13.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=1.56e-02=2^-6 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.196167255915044   +2.024848668626941 *
* i= 2:   -0.000001972783224  +12.022924299555447 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=1.22e-04=2^-13 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193354359578875   +2.120549729983926 *
* i= 2:   -0.000177571937285   +7.774661119977507 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192488833461612   +2.186406788857280 *
* i= 2:   -0.000768433616287   +6.523013374201452 *
*************************************************
```

```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.192536380284465   +2.237249900143503 *
* i= 2:   -0.001699157978718   +5.887993981453639 *
*************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193027820712588   +2.275013677121834 *
* i= 2:   -0.002740977660019   +5.524321249046611 *
*************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193940773918952   +2.312433030514172 *
* i= 2:   -0.004126273461153   +5.226685182851272 *
*************************************************

End of Braess iteration - limit for number of iteration steps reached
```


iteration step 0
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1


iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
0
0.2
0.4
0.6
0.8
1

### 3.30.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+13/t+13step0_x":
A local extremum has been found at x=  +0.366899949555909:
           x                  err(x)
  ------------------     ------------------
  +0.366899949555909     +0.194474619768699
  +0.000000000000000     -0.196742321864963 (*)
  +1.000000000000000     -0.034989708138751
A zero has been found at   x=  +0.046702604714815
A zero has been found at   x=  +0.744191389004425
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+13/t+13step1_x":
A local extremum has been found at x=  +0.364200533726230:
           x                  err(x)
  ------------------     ------------------
  +0.364200533726230     +0.193540392535115
  +0.000000000000000     -0.196165283131820
  +1.000000000000000     -0.157434534755720
A zero has been found at   x=  +0.046443134085913
A zero has been found at   x=  +0.736968401346086
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+13/t+13step999_x":
A local extremum has been found at x=  +0.340194921095547:
           x                  err(x)
  ------------------     ------------------
  +0.340194921095547     +0.181772704574314
  +0.000000000000000     -0.189814500457799
  +1.000000000000000     -0.190393763286514 (*)
A zero has been found at   x=  +0.043873452869685
A zero has been found at   x=  +0.721474625861271
```

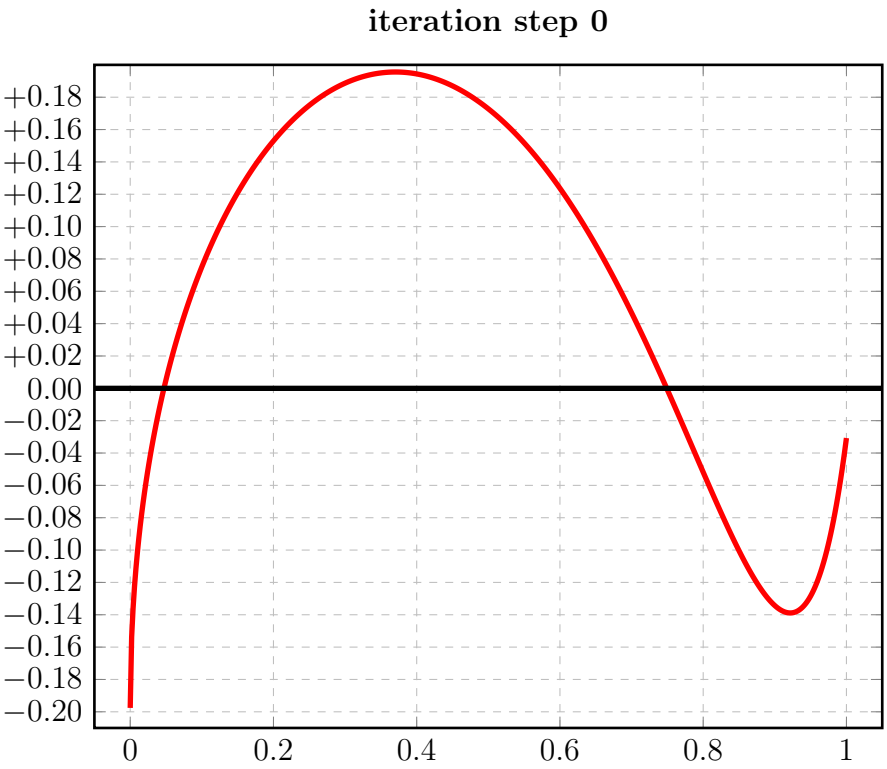
## 3.31 Berechnung für $t_2 = +14.0$

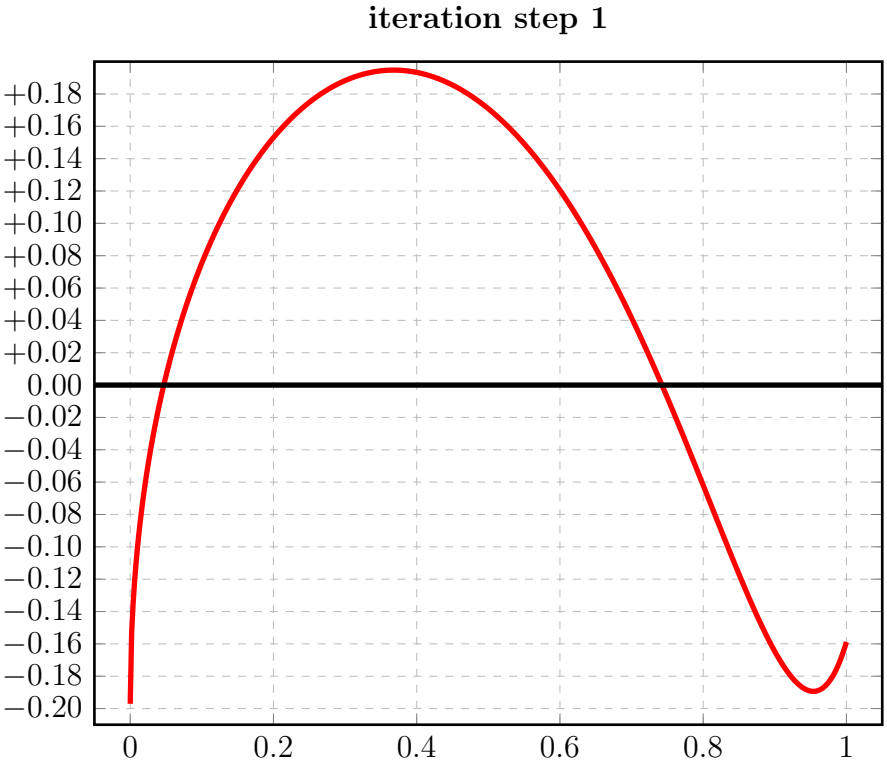
### 3.31.1 Die Berechnung

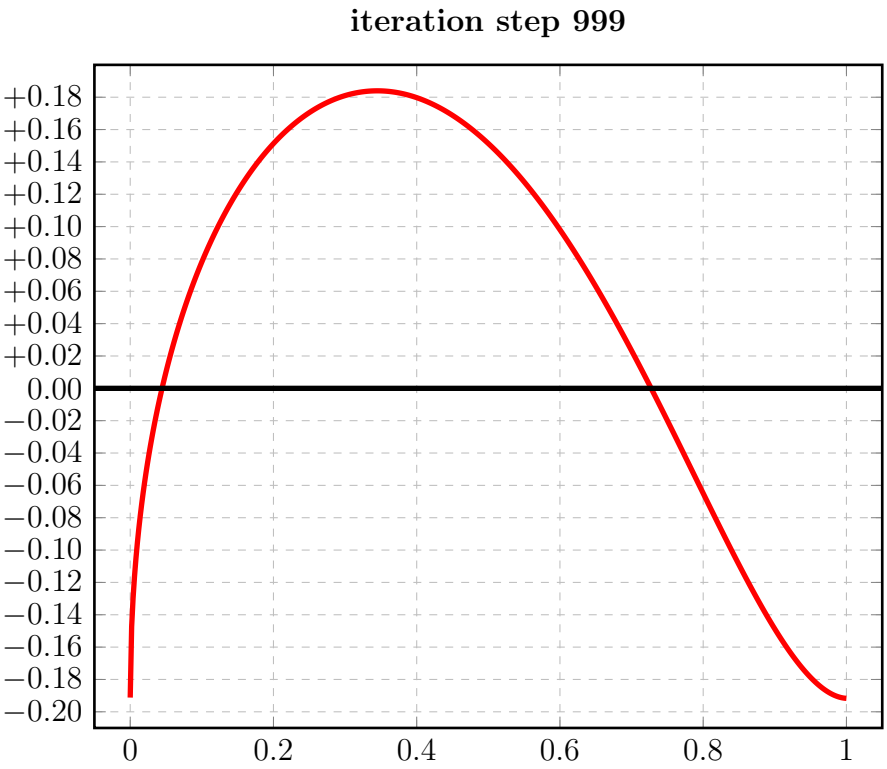
```
Input from job file "../Jobs/abschnitt72/abschnitt72-t+14-terse":
------------------------------------------------------------------
- Function: f(x)=sqrt(x)
- Approximation with respect to V_2
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 0
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.209953239200000   +1.751452157000000
  i= 2:   +0.000000000000000  +14.000000000000000
------------  End Of Initialization  -----------

BRAESS - iteration step 0
Factor c=5.00e-01=2^-1 yields better approximation:
**************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.197589726902359   +1.990925967325178 *
* i= 2:   -0.000000345991617  +14.000000000000000 *
**************************************************
```

```
BRAESS - iteration step 1
Factor c=1.56e-02=2^-6 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.197053512545795   +2.003983141489837 *
* i= 2:   -0.000000719950122  +12.950157917208422 *
*************************************************

-------
OMITTED: iteration steps 2-199
-------

BRAESS - iteration step 200
Factor c=2.44e-04=2^-12 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.194391363498486   +2.086321026506875 *
* i= 2:   -0.000089551765287   +8.340591245982900 *
*************************************************

-------
OMITTED: iteration steps 201 - 399
-------

BRAESS - iteration step 400
Factor c=6.10e-05=2^-14 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193351859571320   +2.141527466507632 *
* i= 2:   -0.000413623135988   +7.000265062327187 *
*************************************************
```

```
-------
OMITTED: iteration steps 401 - 599
-------

BRAESS - iteration step 600
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193035040774136   +2.182678248343088 *
* i= 2:   -0.000921519405760   +6.335738476710715 *
*************************************************

-------
OMITTED: iteration steps 601 - 799
-------

BRAESS - iteration step 800
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193055492311262   +2.208287473363963 *
* i= 2:   -0.001388344472682   +6.008571827765247 *
*************************************************
-------
OMITTED: iteration steps 801 - 998
-------
BRAESS - iteration step 999
Factor c=3.05e-05=2^-15 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.193301296058176   +2.237660189807467 *
* i= 2:   -0.002091869181510   +5.691355182280232 *
*************************************************

End of Braess iteration - limit for number of iteration steps reached
```

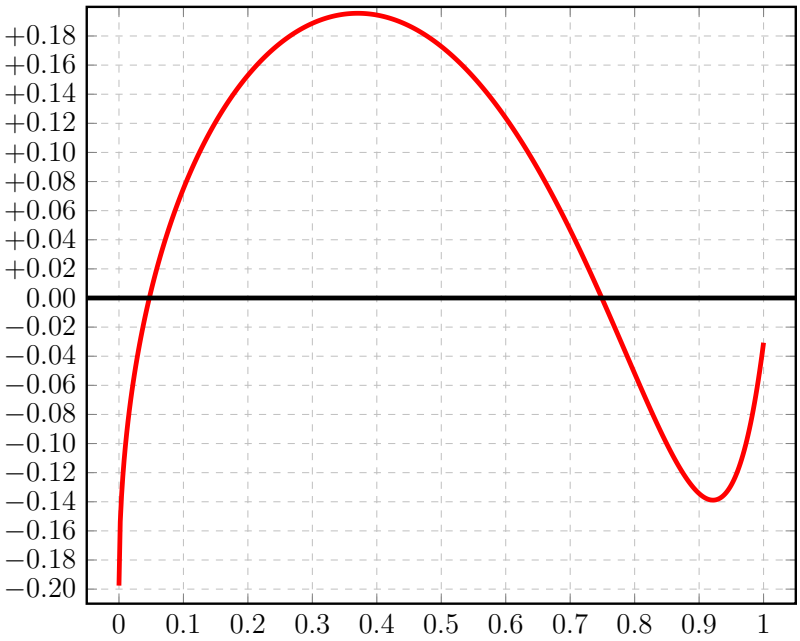

iteration step 0
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1


iteration step 1
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

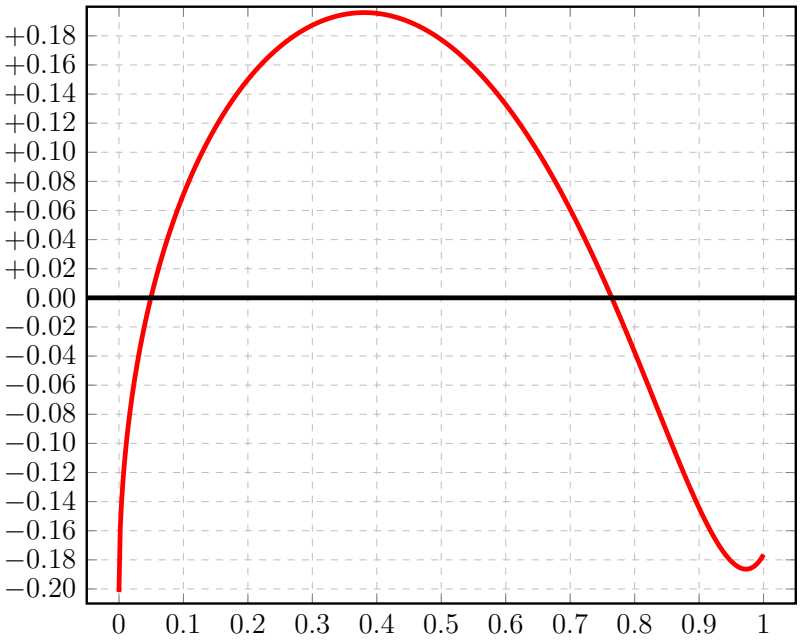

iteration step 999
+0.18
+0.16
+0.14
+0.12
+0.10
+0.08
+0.06
+0.04
+0.02
0.00
−0.02
−0.04
−0.06
−0.08
−0.10
−0.12
−0.14
−0.16
−0.18
−0.20
0
0.2
0.4
0.6
0.8
1

Zum Vergleich: die Startfunktionen von [1] nach Fall 2 (Zeichnung 14E) und Fall 1 (Zeichnung 14J):

**Zeichnung 14E von [1], §7.2**




**Zeichnung 14J von [1], §7.2**



### 3.31.2 Auswertungen mit EXPAPP_EVAL_0

**Auswertung Iteration Step 0:**

```
Input from job file "../Jobs/para72/t+14/t+14step0_x"
A local extremum has been found at x=  +0.370404608977680:
           x                  err(x)
  ------------------     ------------------
  +0.370404608977680     +0.195590710896128
  +0.000000000000000     -0.197589380910742 (*)
  +1.000000000000000     -0.030722399215857
A zero has been found at   x=  +0.047093992790942
A zero has been found at   x=  +0.749057935902931
```

**Auswertung Iteration Step 1:**

```
Input from job file "../Jobs/para72/t+14/t+14step1_x"
A local extremum has been found at x=  +0.367908367147035:
           x                  err(x)
  ------------------     ------------------
  +0.367908367147035     +0.194748450229047
  +0.000000000000000     -0.197052792595673
  +1.000000000000000     -0.158821398178627
A zero has been found at   x=  +0.046850310779706
A zero has been found at   x=  +0.742421291231528
```

**Auswertung Iteration Step 999:**

```
Input from job file "../Jobs/para72/t+14/t+14step999_x"
A local extremum has been found at x=  +0.345007548160317:
           x                  err(x)
  ------------------     ------------------
  +0.345007548160317     +0.183951803802572
  +0.000000000000000     -0.191209426876666
  +1.000000000000000     -0.191689451543578 (*)
A zero has been found at   x=  +0.044445168858834
A zero has been found at   x=  +0.727336750868146
```

## 3.32 Zusammenfassung, Vergleich

Zu: **Startwerte $\mathbf{t_2} \in [-15, +1]$**

Mit Startwerten $t_2 \in [-15, +1]$ berechnet BRAESS erfolgreich eine "best approximation". Tabelle 3-1 enthält für diese Startwerte

- die für die Berechnung der jeweiligen "best approximation" erforderliche Zahl von Iterationsschritten
- die Norm der Fehlerfunktion von "iteration step 0".
  Das Minimum der Norm der Fehlerfunktion erhält man bei diesen Berechnungen bei $t_2 = -8$.[4]
- die Norm der Fehlerfunktion der "best approximation"
- relative Abweichung der Alternante der "best approximation".

Signifikante Abweichungen gegenüber [1] sind bei den Startfunktionen mit den $t_2$-Werten -11 bis -6 und +2 festzustellen. *Wiederholung dieser Berechnungen mit kleineren Werten für $\delta_R$, $\delta_B$!*

Sonst:
Es gelten weitgehend die Ausführungen in [1], §7, zu "Abschnitt 7.3", insbesondere auch bzgl. "Vorzeichenklassen".

[4] die Berechnungen in [1], §7, ergaben ein Minimum bei $t_2 = -10$

| $\mathbf{t_2}$ | $\delta_{\mathbf{R}}$ | $\delta_{\mathbf{B}}$ | **itera-tions** | **norm of error function** iteration step 0 | **norm of error function** best approximation | **relative deviation** |
|---|---|---|---|---|---|---|
| -15 | 0.01 | 0.01 | 4 | 0.149018547847034 | 0.030190865460118 | 0.011926 |
| -15 | 0.001 | 0.001 | 4 | 0.149009215899198 | 0.030206317120479 | 0.011731 |
| -14 | 0.01 | 0.01 | 4 | 0.148492912675583 | 0.030190866800890 | 0.011925 |
| -13 | 0.01 | 0.01 | 4 | 0.147993198537862 | 0.030191098039131 | 0.011930 |
| -12 | 0.01 | 0.01 | 3 | 0.151661109503686 | 0.030185425056236 | 0.014300 |
| -11 | 0.01 | 0.01 | 3 | 0.149771201137898 | 0.030190919741009 | 0.011930 |
| -10 | 0.01 | 0.01 | 3 | 0.148047181239196 | 0.030190835924154 | 0.011926 |
| -9 | 0.01 | 0.01 | 3 | 0.146971002524375 | 0.030190588325803 | 0.011921 |
| -8 | 0.01 | 0.01 | 3 | 0.146740827271900 | 0.030190299038958 | 0.012032 |
| -7 | 0.01 | 0.01 | 3 | 0.147874590729322 | 0.030191952003671 | 0.012148 |
| -6 | 0.01 | 0.01 | 4 | 0.150830093209882 | 0.030190841671511 | 0.011931 |
| -5 | 0.01 | 0.01 | 4 | 0.156366067711090 | 0.030190684735756 | 0.011967 |
| -4 | 0.01 | 0.01 | 4 | 0.166653412199856 | 0.030187887081252 | 0.012903 |
| -3 | 0.01 | 0.01 | 6 | 0.174172850600343 | 0.030191271980893 | 0.011934 |
| -2 | 0.01 | 0.01 | 6 | 0.185445188878440 | 0.030192437466439 | 0.011360 |
| -1 | 0.01 | 0.01 | 10 | 0.197505163901540 | 0.030214000410553 | 0.015683 |
| 0 | 0.01 | 0.01 | 22 | 0.207092775191915 | 0.030189552435806 | 0.012443 |
| +1 | 0.01 | 0.01 | 95 | 0.209573291590398 | 0.030190961938546 | 0.011933 |

**Tabelle 3-1**

Zu: **Startwerte $\mathbf{t_2} \geq +2$**

Für die Startwerte $t_2 \geq +2$ wird auch nach 999 Iterationsschritten keine "best approximation" erreicht, s. Tabelle 3-2. Man vergleiche hierzu auch die Ausführungen in [1], §7, zu "Abschnitt 7.3".

$\Delta_{norm}$ von Tabelle 3-2 gibt die Differenz der Normen an [5].

| $\mathbf{t_2}$ | $\delta_\mathbf{R}$ | $\delta_\mathbf{B}$ | **iterations** | **norm of error function iteration step 0** | **norm of error function iteration step 999** | $\Delta_{norm}$ |
|---|---|---|---|---|---|---|
| +2 | 0.01 | 0.01 | 999 | 0.209947971029700 | 0.209947957567539 | 0.000 000 0135 |
| +3 | 0.01 | 0.01 | 999 | 0.209362259294687 | 0.209335436245077 | 0.000 027 |
| +4 | 0.01 | 0.01 | 999 | 0.205825537029951 | 0.205597781352963 | 0.000 228 |
| +5 | 0.01 | 0.01 | 999 | 0.202698353304186 | 0.202504878223934 | 0.000 194 |
| +6 | 0.01 | 0.01 | 999 | 0.197118354461715 | 0.195598414052391 | 0.001 520 |
| +7 | 0.01 | 0.01 | 999 | 0.187074044732651 | 0.181844341247895 | 0.005 230 |
| +8 | 0.01 | 0.01 | 999 | 0.189730667445087 | 0.183349226201721 | 0.006 381 |
| +9 | 0.01 | 0.01 | 999 | 0.191675969459490 | 0.184557006517380 | 0.007 118 |
| +10 | 0.01 | 0.01 | 999 | 0.193285870725733 | 0.185916413037059 | 0.007 369 |
| +11 | 0.01 | 0.01 | 999 | 0.194625902826534 | 0.188186897441108 | 0.006 439 |
| +12 | 0.01 | 0.01 | 999 | 0.195780234303229 | 0.189214375479192 | 0.006 566 |
| +13 | 0.01 | 0.01 | 999 | 0.196742321864963 | 0.190393763286514 | 0.006 349 |
| +14 | 0.01 | 0.01 | 999 | 0.197589380910742 | 0.191689451543578 | 0.005 900 |

**Tabelle 3-2**

[5] "step 0" - "step 999"

# 4 Approximationen zu $\mathbf{f(x) = \frac{1}{1+x}}$ bzgl. $\mathbf{V_3^0}$

Zwei Berechnungen von [1], §7.4 zu "Konvergenzverhalten":
Die Norm der Startfunktionen lässt keine Aussagen zur Zahl der erforderlichen Iterationsschritte zu.

Für diese Berechnungen wurden 2 Startfunktionen mit MEINARDUS [7] berechnet:

- Berechnung 1 (Abschnitt 4.1): s. [8], Abschnitt 1.1
- Berechnung 2 (Abschnitt 4.2): s. [8], Abschnitt 1.2

## 4.1 Berechnung 1 mit $\delta_R = 0.01$ $\delta_B = 0.01$

### 4.1.1 Die Berechnung

```
Input from job file "../Jobs/para74/para74a-terse-plots":
-------------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.546271421802803   -0.277576311531820
  i= 2:   +0.401383367280923   -1.546985019769484
  i= 3:   +0.052345210916276   -4.336157075160293
------------  End Of Initialization  -----------
```

```
BRAESS - iteration step 1

The parameters r[i] of linear approximation:
r[0]=  +0.015503888611545
r[1]=  -0.008618415223276
r[2]=  -0.006887325892998
r[3]=  -0.010865694516245
r[4]=  -0.060992088209056
r[5]=  -0.166810565070660

Factor c=6.25e-02=2^-4 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.547240414841025   -0.278255417439085 *
* i= 2:   +0.400844716329468   -1.550797025282550 *
* i= 3:   +0.051914753047964   -4.346582735477210 *
*************************************************

BRAESS - iteration step 2

The parameters r[i] of linear approximation:
r[0]=  +0.014350885957775
r[1]=  -0.007956621613546
r[2]=  -0.006395998344443
r[3]=  -0.010059072224746
r[4]=  -0.056748795722660
r[5]=  -0.156788942542556

Factor c=6.25e-02=2^-4 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.548137345213386   -0.278884109453132 *
* i= 2:   +0.400347427478622   -1.554343825015216 *
* i= 3:   +0.051515003151436   -4.356382044386120 *
*************************************************
```

```
BRAESS - iteration step 3

The parameters r[i] of linear approximation:
r[0]=  +0.013294746203778
r[1]=  -0.007353403247148
r[2]=  -0.005942966191798
r[3]=  -0.009320150304695
r[4]=  -0.052824571914766
r[5]=  -0.147339686479900

Factor c=6.25e-02=2^-4 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.548968266851122   -0.279466618847175 *
* i= 2:   +0.399887839775675   -1.557645360759889 *
* i= 3:   +0.051143567764449   -4.365590774791113 *
*************************************************

BRAESS - iteration step 4

The parameters r[i] of linear approximation:
r[0]=  +0.012325935400320
r[1]=  -0.006802568742677
r[2]=  -0.005524886342671
r[3]=  -0.008642250224582
r[4]=  -0.049192701355128
r[5]=  -0.138434450219436

Factor c=6.25e-02=2^-4 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.549738637813642   -0.280006759486212 *
* i= 2:   +0.399462679229257   -1.560719904594585 *
* i= 3:   +0.050798262368032   -4.374242927929828 *
*************************************************
```

```
BRAESS - iteration step 5

The parameters r[i] of linear approximation:
r[0]=  +0.011436028810553
r[1]=  -0.006298708435640
r[2]=  -0.005138743237959
r[3]=  -0.008019476626974
r[4]=  -0.045828993220561
r[5]=  -0.130045604162959

Factor c=6.25e-02=2^-4 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.550453389614301   -0.280507976775398 *
* i= 2:   +0.399069009952030   -1.563584216670870 *
* i= 3:   +0.050477090915659   -4.382370778190013 *
*************************************************

BRAESS - iteration step 6

The parameters r[i] of linear approximation:
r[0]=  +0.010617561330936
r[1]=  -0.005837079793568
r[2]=  -0.004781813854671
r[3]=  -0.007446610115645
r[4]=  -0.042711516399315
r[5]=  -0.122146345502964

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.551780584780668   -0.281438803039853 *
* i= 2:   +0.398339374977834   -1.568923156220784 *
* i= 3:   +0.049879364183825   -4.397639071377883 *
*************************************************
```

```
BRAESS - iteration step 7

The parameters r[i] of linear approximation:
r[0]=  +0.009116925402441
r[1]=  -0.004994361583660
r[2]=  -0.004123726849927
r[3]=  -0.006396026195353
r[4]=  -0.036945032921882
r[5]=  -0.107267367334318

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.552920200455973   -0.282238306314272 *
* i= 2:   +0.397715079779877   -1.573541285336019 *
* i= 3:   +0.049363898327584   -4.411047492294673 *
*************************************************

BRAESS - iteration step 8

The parameters r[i] of linear approximation:
r[0]=  +0.007847419442848
r[1]=  -0.004286121492607
r[2]=  -0.003562313503988
r[3]=  -0.005506944751406
r[4]=  -0.032001553841516
r[5]=  -0.094148420228656

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.553901127886329   -0.282926674408198 *
* i= 2:   +0.397179314593301   -1.577541479566209 *
* i= 3:   +0.048918609139586   -4.422816044823255 *
*************************************************
```

```
BRAESS - iteration step 9

The parameters r[i] of linear approximation:
r[0]=  +0.006768947095564
r[1]=  -0.003687798440806
r[2]=  -0.003082035660570
r[3]=  -0.004751372359978
r[4]=  -0.027753658251692
r[5]=  -0.082595736864812

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.554747246273275   -0.283520595953196 *
* i= 2:   +0.396718339788200   -1.581010686847670 *
* i= 3:   +0.048533354682015   -4.433140511931356 *
*************************************************

BRAESS - iteration step 10

The parameters r[i] of linear approximation:
r[0]=  +0.005849396776965
r[1]=  -0.003180044843270
r[2]=  -0.002670126837965
r[3]=  -0.004106901096539
r[4]=  -0.024095773726639
r[5]=  -0.072432687987443

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.555478420870395   -0.284033958590263 *
* i= 2:   +0.396320834182791   -1.584022658563500 *
* i= 3:   +0.048199588827269   -4.442194597929786 *
*************************************************
```

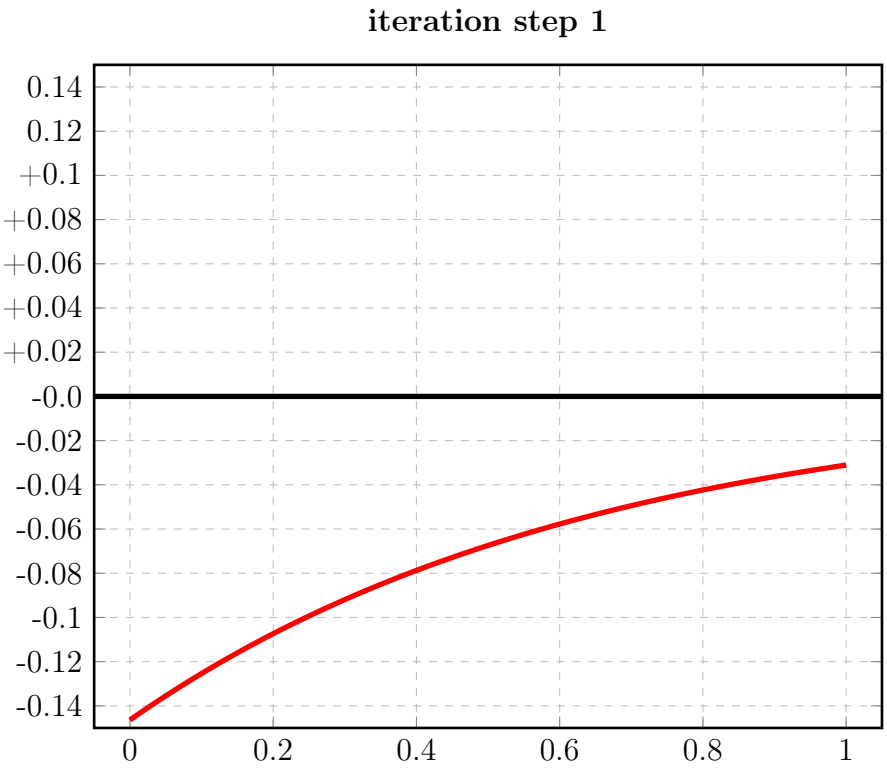
```
BRAESS - iteration step 11

The parameters r[i] of linear approximation:
r[0]=  +0.005062825234582
r[1]=  -0.002747455231410
r[2]=  -0.002316047107281
r[3]=  -0.003555430044988
r[4]=  -0.020940014612594
r[5]=  -0.063499606233880

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.556111274024718   -0.284478387345886 *
* i= 2:   +0.395977402278865   -1.586640160390074 *
* i= 3:   +0.047910082938859   -4.450132048709022 *
*************************************************

BRAESS - iteration step 12

The parameters r[i] of linear approximation:
r[0]=  +0.004388103161993
r[1]=  -0.002377641849729
r[2]=  -0.002011053061817
r[3]=  -0.003082215601705
r[4]=  -0.018212884352332
r[5]=  -0.055653071611875

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.556659786919967   -0.284863664296099 *
* i= 2:   +0.395680197047649   -1.588916770934116 *
* i= 3:   +0.047658701306132   -4.457088682660506 *
*************************************************
```

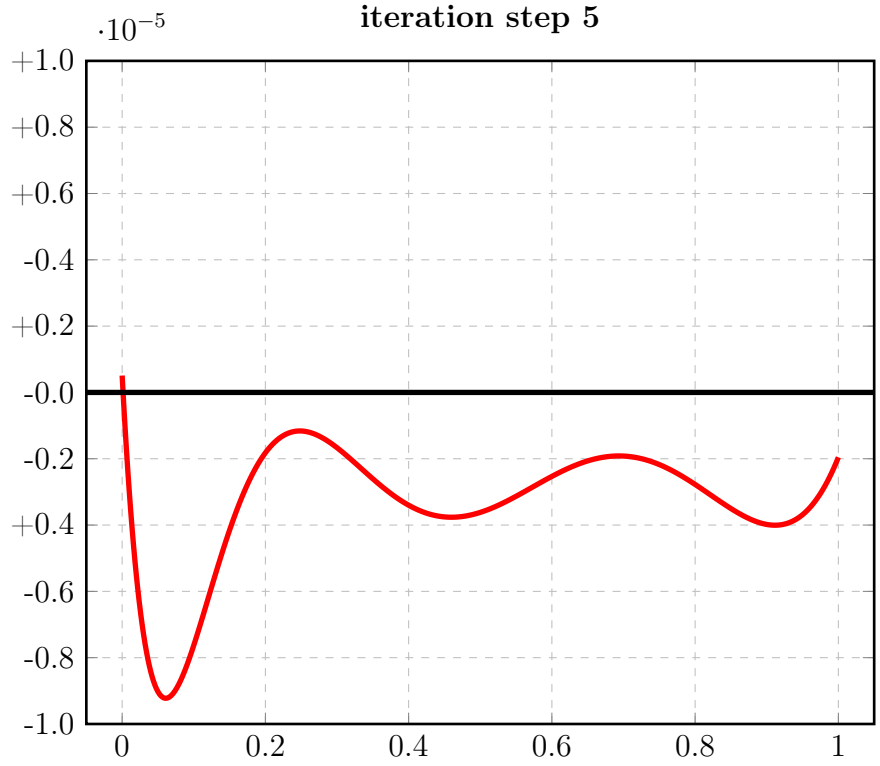
```
BRAESS - iteration step 13

The parameters r[i] of linear approximation:
r[0]=  +0.003807894473585
r[1]=  -0.002060553892612
r[2]=  -0.001747857813874
r[3]=  -0.002675157746685
r[4]=  -0.015852653017301
r[5]=  -0.048764889203211

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.557135773729166   -0.285198059014435 *
* i= 2:   +0.395422627811072   -1.590898352561279 *
* i= 3:   +0.047440219079398   -4.463184293810907 *
*************************************************

BRAESS - iteration step 14

The parameters r[i] of linear approximation:
r[0]=  +0.003307879144364
r[1]=  -0.001787970023292
r[2]=  -0.001520361280766
r[3]=  -0.002324257732801
r[4]=  -0.013807262690680
r[5]=  -0.042720912637291

Factor c=1.25e-01=2^-3 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.557549258622211   -0.285488591231035 *
* i= 2:   +0.395199131558161   -1.592624260397614 *
* i= 3:   +0.047250173919302   -4.468524407890569 *
*************************************************
```

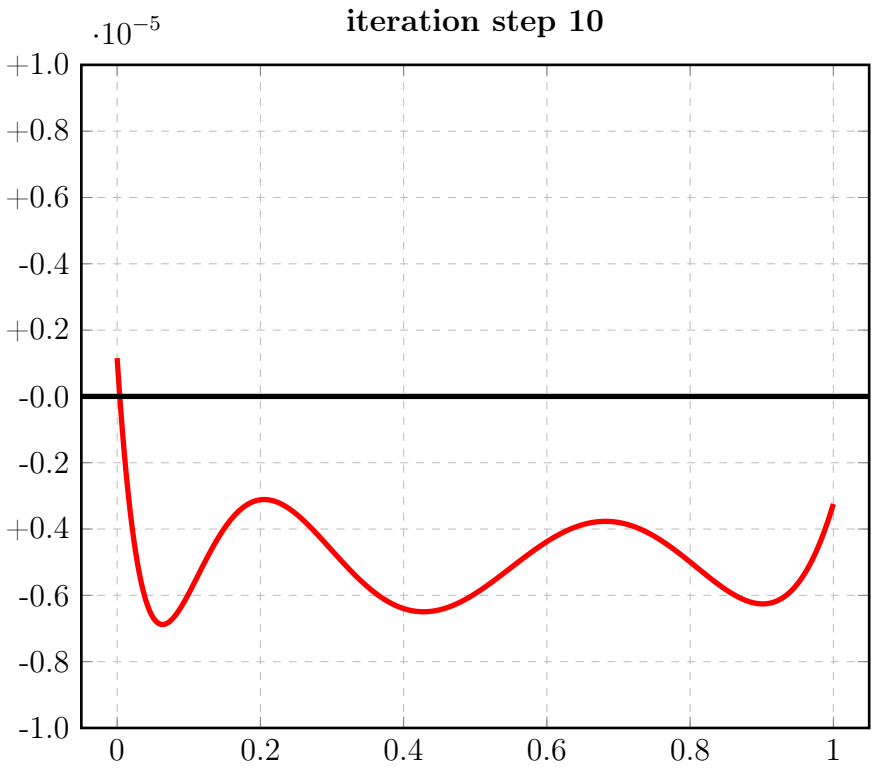
```
BRAESS - iteration step 15

The parameters r[i] of linear approximation:
r[0]=  +0.002876155689853
r[1]=  -0.001553115651928
r[2]=  -0.001323435357086
r[3]=  -0.002021201725810
r[4]=  -0.012032646293130
r[5]=  -0.037419813613924

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.558268297544674   -0.285993891662488 *
* i= 2:   +0.394810852645179   -1.595632421970896 *
* i= 3:   +0.046919315080030   -4.477879361294050 *
*************************************************

BRAESS - iteration step 16

The parameters r[i] of linear approximation:
r[0]=  +0.002131320915997
r[1]=  -0.001148824593155
r[2]=  -0.000982792354536
r[3]=  -0.001498192312442
r[4]=  -0.008955193857033
r[5]=  -0.028120391303051

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.558801127773673   -0.286368439740598 *
* i= 2:   +0.394523646496890   -1.597871220435155 *
* i= 3:   +0.046673616991396   -4.484909459119812 *
*************************************************
```

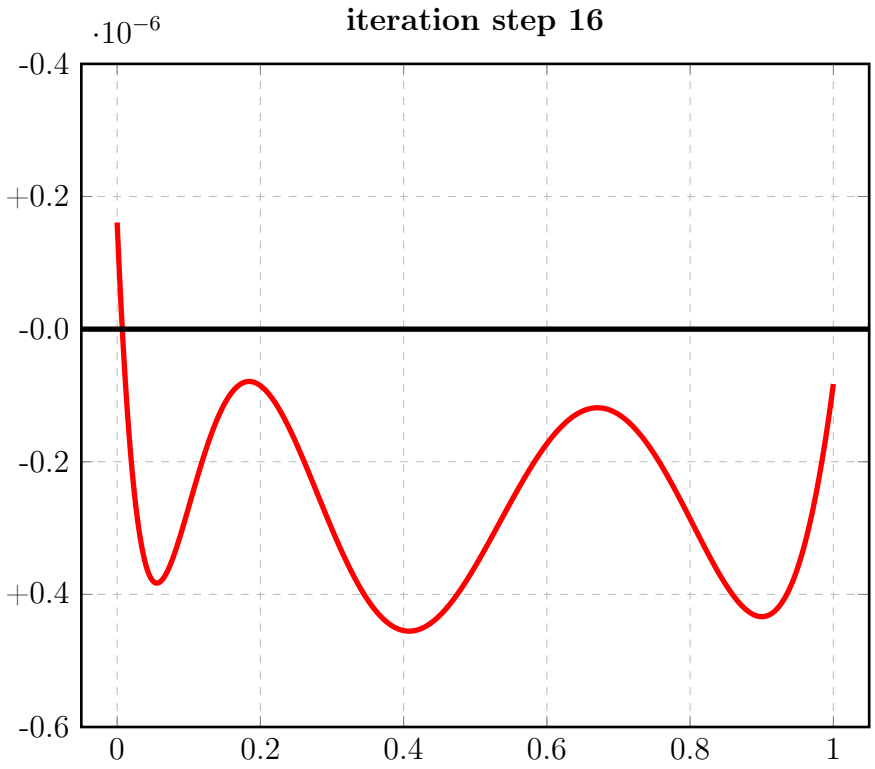
```
BRAESS - iteration step 17

The parameters r[i] of linear approximation:
r[0]=  +0.001584165047494
r[1]=  -0.000852778118018
r[2]=  -0.000731608696186
r[3]=  -0.001113816157334
r[4]=  -0.006677555988720
r[5]=  -0.021120529681984

Factor c=5.00e-01=2^-1 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.559593210297420   -0.286925347819265 *
* i= 2:   +0.394097257437881   -1.601209998429515 *
* i= 3:   +0.046307812643303   -4.495469723960804 *
*************************************************

BRAESS - iteration step 18

The parameters r[i] of linear approximation:
r[0]=  +0.000778441069964
r[1]=  -0.000418012161081
r[2]=  -0.000360539544710
r[3]=  -0.000547553692943
r[4]=  -0.003301496076011
r[5]=  -0.010588831197112

Factor c=1.00e+00=2^-0 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.560371651367385   -0.287472901512208 *
* i= 2:   +0.393679245276800   -1.604511494505525 *
* i= 3:   +0.045947273098594   -4.506058555157916 *
*************************************************
```

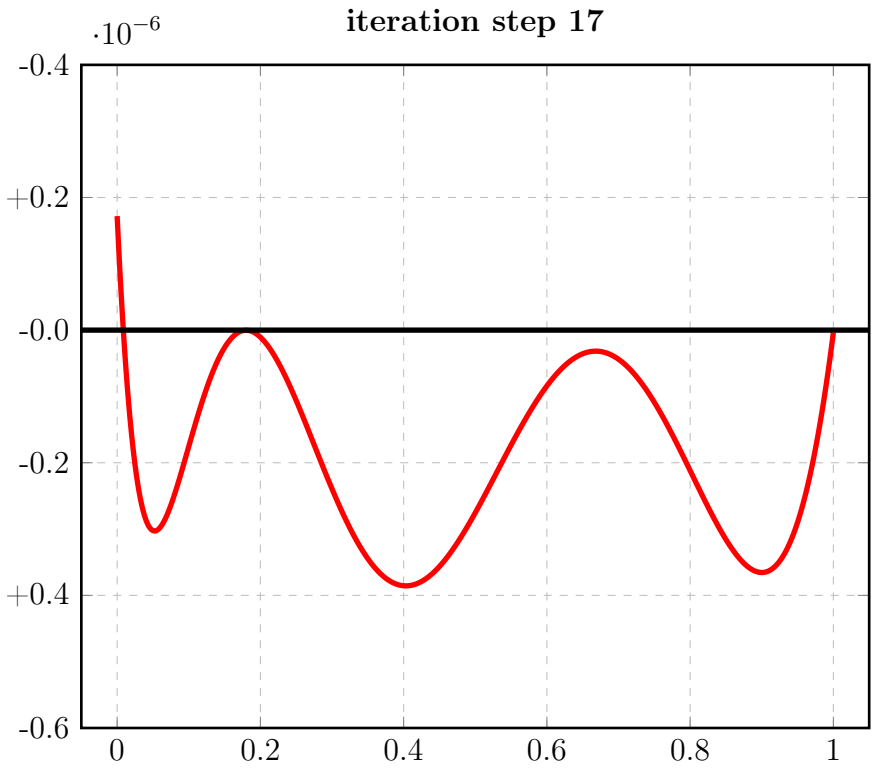
```
BRAESS - iteration step 19

The parameters r[i] of linear approximation:
r[0]=  -0.000004473736166
r[1]=  +0.000002718172832
r[2]=  +0.000001755645644
r[3]=  +0.000003063391136
r[4]=  +0.000012413141548
r[5]=  -0.000008646041011

Factor c=1.00e+00=2^-0 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.560367177631219   -0.287469838121072 *
* i= 2:   +0.393681963449633   -1.604499081363977 *
* i= 3:   +0.045949028744238   -4.506067201198927 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```

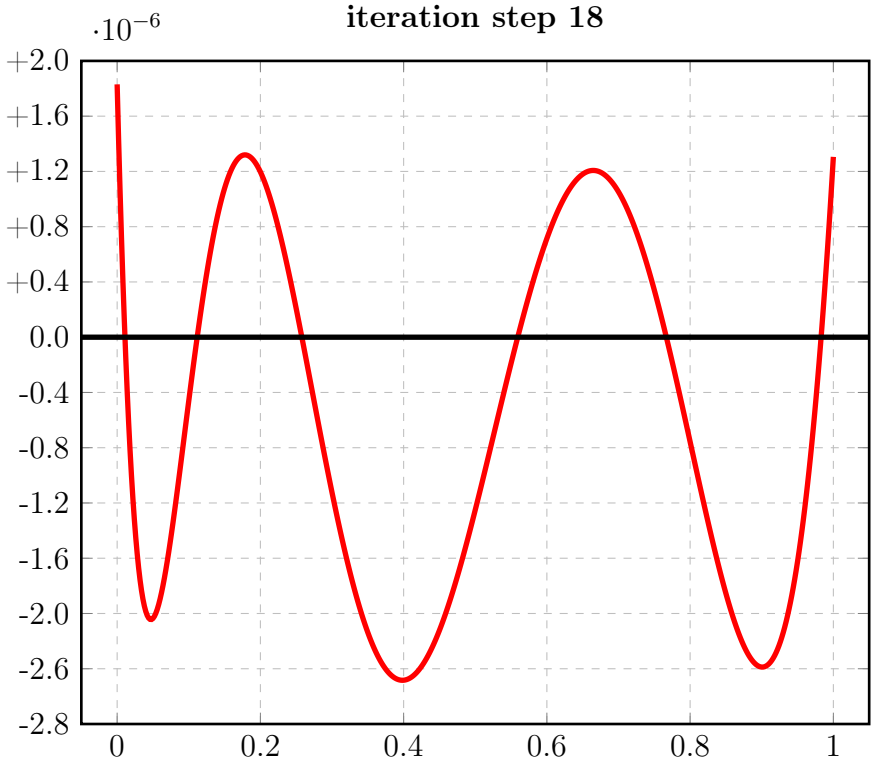

iteration step 1
0.14
0.12
+0.1
+0.08
+0.06
+0.04
+0.02
-0.0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
-0.14
0
0.2
0.4
0.6
0.8
1

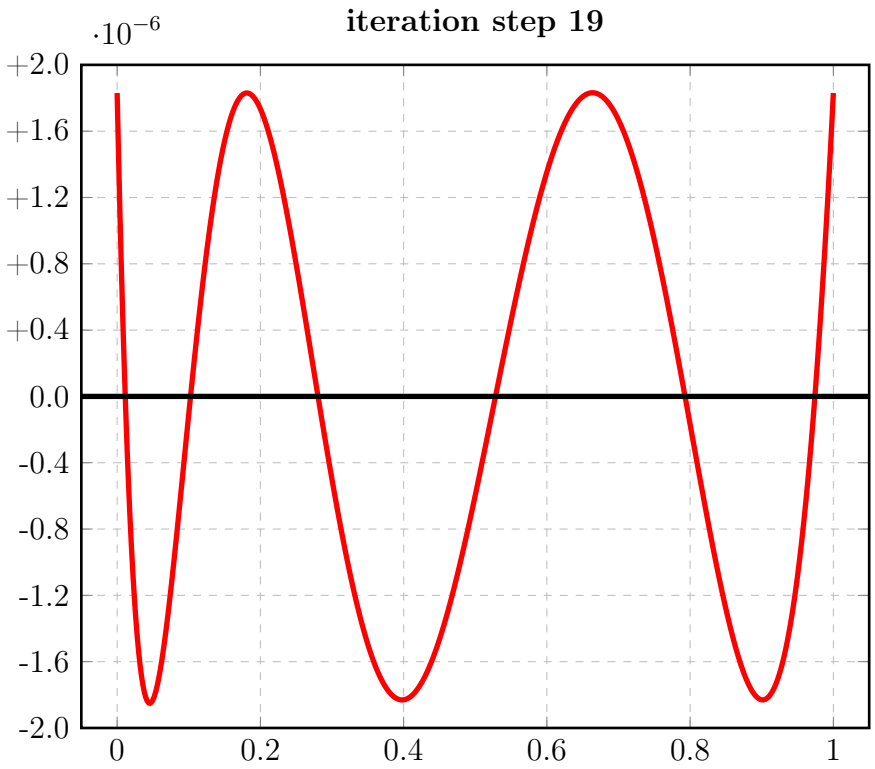

iteration step 5
$\cdot 10^{-5}$
+1.0
+0.8
+0.6
-0.4
+0.2
-0.0
-0.2
+0.4
-0.6
-0.8
-1.0
0
0.2
0.4
0.6
0.8
1


iteration step 10
$\cdot 10^{-5}$
+1.0
+0.8
+0.6
-0.4
+0.2
-0.0
-0.2
+0.4
-0.6
-0.8
-1.0
0
0.2
0.4
0.6
0.8
1


iteration step 16
$\cdot 10^{-6}$
-0.4
+0.2
-0.0
-0.2
+0.4
-0.6
0
0.2
0.4
0.6
0.8
1


iteration step 17
$\cdot 10^{-6}$
-0.4
+0.2
-0.0
-0.2
+0.4
-0.6
0
0.2
0.4
0.6
0.8
1


iteration step 18
$\cdot 10^{-6}$
+2.0
+1.6
+1.2
+0.8
+0.4
0.0
-0.4
-0.8
-1.2
-1.6
-2.0
-2.6
-2.8
0
0.2
0.4
0.6
0.8
1


iteration step 19
$\cdot 10^{-6}$
+2.0
+1.6
+1.2
+0.8
+0.4
0.0
-0.4
-0.8
-1.2
-1.6
-2.0
0
0.2
0.4
0.6
0.8
1

### 4.1.2 Auswertungen

**Auswertung Iteration Step 1**

```
Input from job file "../Jobs/para74/Berechnung1/para74-1-step1_x1":

Local extremum have been found:
           x                  err(x)
  ------------------     ------------------
  +0.059174835997985     -0.000010455220906 (*)
  +0.269041896767963     +0.000000881899604
  +0.702563711707355     -0.000000170708775
  +0.921168222879959     -0.000002097685119
  +0.000000000000000     +0.000000115781543
  +1.000000000000000     -0.000000633143429

A zero has been found at   x=  +0.000260086456338
A zero has been found at   x=  +0.214311351451672
A zero has been found at   x=  +0.343302837368865
```

**Auswertung Iteration Step 19**

```
Input from job file "../Jobs/para74/para74-1-step19":

The exact local extrema of f-E(a) in interval I:
-----------------------------------------------
               x[i]                  y[i]
         ------------------   ------------------
i= 0:   +0.000000000000000   +0.000001830174910
i= 1:   +0.045495536302989   -0.000001851393784
i= 2:   +0.181070047454212   +0.000001830497980
i= 3:   +0.397873935730943   -0.000001830831649
i= 4:   +0.663792957311643   +0.000001831964619
i= 5:   +0.901320287620686   -0.000001830632375
i= 6:   +1.000000000000000   +0.000001830170490

Norm of error function:       0.000001851393784
Relative deviation:           0.011463414194828
```

```
The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.011294964283044
i= 1:   +0.102467140015567
i= 2:   +0.280468604544167
i= 3:   +0.528287834271747
i= 4:   +0.792912825247682
i= 5:   +0.974224734484426
```

## 4.2 Berechnung 2 mit $\delta_{\mathbf{R}} = 0.01$ $\delta_{\mathbf{B}} = 0.01$

### 4.2.1 Die Berechnung

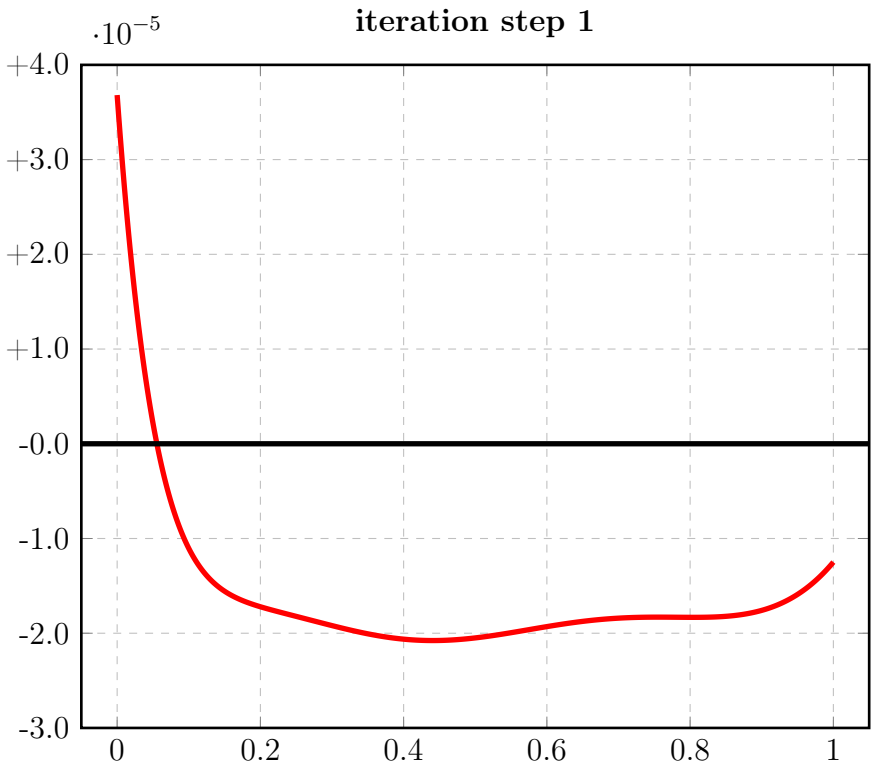
```
Input from job file "../Jobs/para74/para74b-terse-plots":
------------------------------------------------------------
- Function: f(x)=1/(1+x)
- Approximation with respect to V_3
- Interval                      : [0.00,1.00]
- Distance of equidistant points: 0.0100
- Braess termination criterion  : 1.000000E-02
- Remez  termination criterion  : 1.000000E-02
- Plot-Indicator                : 1
- output                        : terse
- Starting parameters:
                a[i]                t[i]
  i= 1:   +0.545644214548501   -0.277434496518055
  i= 2:   +0.400482842986620   -1.540603343594550
  i= 3:   +0.053824480225551   -4.282493872762781
------------  End Of Initialization  -----------
```

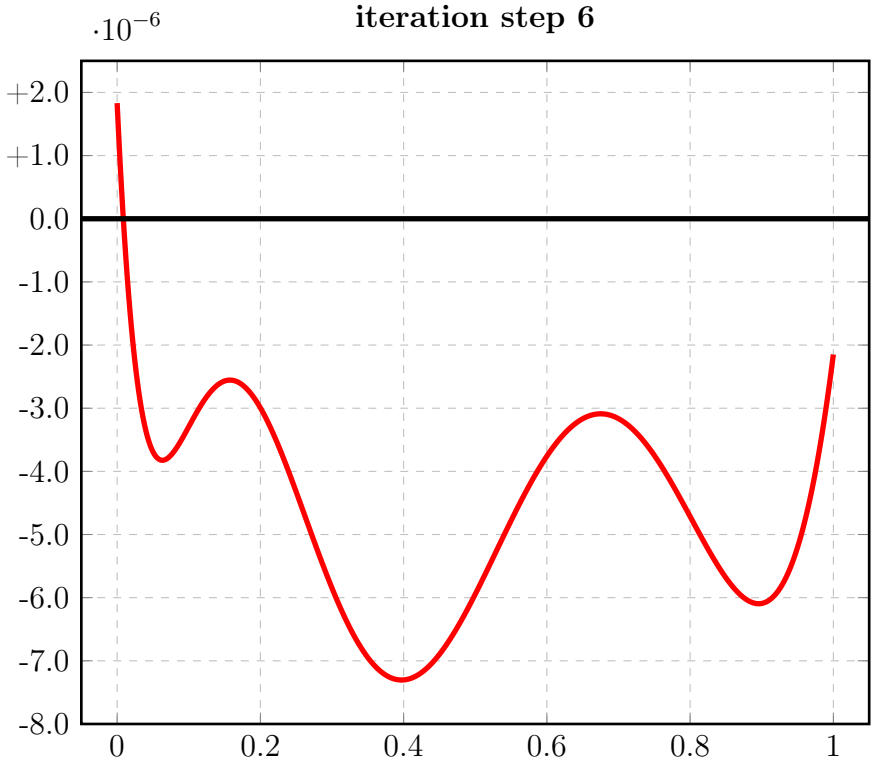
```
BRAESS - iteration step 1

The parameters r[i] of linear approximation:
r[0]=  +0.016951655728720
r[1]=  -0.008015283689588
r[2]=  -0.008889775575687
r[3]=  -0.011528882945118
r[4]=  -0.070738478057876
r[5]=  -0.223384937674888

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.549882128480681   -0.280316717254335 *
* i= 2:   +0.398479022064223   -1.558287963109019 *
* i= 3:   +0.051602036331629   -4.338340107181503 *
*************************************************

BRAESS - iteration step 2

The parameters r[i] of linear approximation:
r[0]=  +0.011649677160790
r[1]=  -0.005410660892403
r[2]=  -0.006204052979365
r[3]=  -0.007932048817870
r[4]=  -0.049924498561814
r[5]=  -0.168025868442104

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.552794547770879   -0.282299729458802 *
* i= 2:   +0.397126356841122   -1.570769087749472 *
* i= 3:   +0.050051023086788   -4.380346574292029 *
*************************************************
```

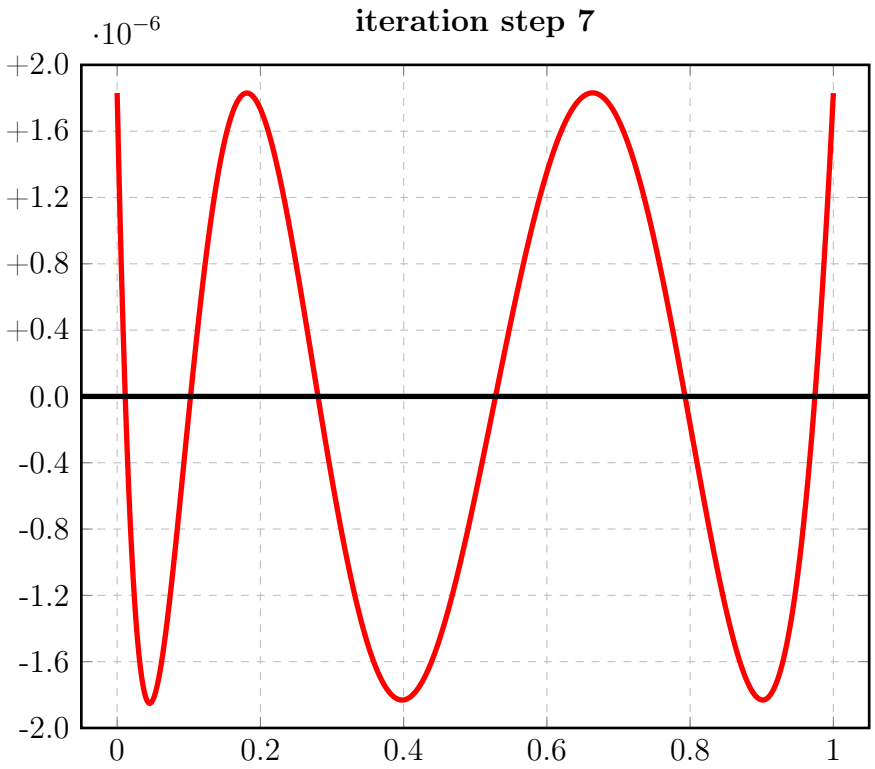
```
BRAESS - iteration step 3

The parameters r[i] of linear approximation:
r[0]=  +0.008192835354011
r[1]=  -0.003763459168432
r[2]=  -0.004403144956766
r[3]=  -0.005584474590492
r[4]=  -0.035759201084009
r[5]=  -0.126049427074803

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.554842756609381   -0.283695848106425 *
* i= 2:   +0.396185492049014   -1.579708888020475 *
* i= 3:   +0.048950236847597   -4.411858931060730 *
*************************************************

BRAESS - iteration step 4

The parameters r[i] of linear approximation:
r[0]=  +0.005859366275511
r[1]=  -0.002672931568762
r[2]=  -0.003166756447805
r[3]=  -0.003997636133012
r[4]=  -0.025906036425297
r[5]=  -0.094457607369100

Factor c=2.50e-01=2^-2 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.556307598178259   -0.284695257139678 *
* i= 2:   +0.395517259156824   -1.586185397126799 *
* i= 3:   +0.048158547735645   -4.435473332903006 *
*************************************************
```

```
BRAESS - iteration step 5
-------------------------

Factor c=5.00e-01=2^-1 yields better approximation:
***************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.558428777012869   -0.286143547408795 *
* i= 2:   +0.394553954070072   -1.595650630617565 *
* i= 3:   +0.047008054675489   -4.470853412041181 *
***************************************************

BRAESS - iteration step 6

The parameters r[i] of linear approximation:
r[0]=  +0.001981694383902
r[1]=  -0.000892974719521
r[2]=  -0.001081336426609
r[3]=  -0.001355144848200
r[4]=  -0.008998504255906
r[5]=  -0.035277621120757

Factor c=1.00e+00=2^-0 yields better approximation:
***************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.560410471396771   -0.287498692256995 *
* i= 2:   +0.393660979350551   -1.604649134873471 *
* i= 3:   +0.045926718248880   -4.506131033161938 *
***************************************************
```

```
BRAESS - iteration step 7

The parameters r[i] of linear approximation:
r[0]=  -0.000043289965178
r[1]=  +0.000020979081953
r[2]=  +0.000022311712065
r[3]=  +0.000028850946650
r[4]=  +0.000150056429058
r[5]=  +0.000063886091797

Factor c=1.00e+00=2^-0 yields better approximation:
*************************************************
* BRAESS-iteration terminated with approximation: *
*                  a[i]                 t[i]      *
*         ------------------   ------------------ *
* i= 1:   +0.560367181431594   -0.287469841310345 *
* i= 2:   +0.393681958432504   -1.604499078444414 *
* i= 3:   +0.045949029960944   -4.506067147070141 *
*************************************************

              END OF Braess ITERATION:
       a best approximation has been calculated
             with delta_B = 1.000000E-02
```


iteration step 1
$\cdot 10^{-5}$
+4.0
+3.0
+2.0
+1.0
-0.0
-1.0
-2.0
-3.0
0
0.2
0.4
0.6
0.8
1


iteration step 6
$\cdot 10^{-6}$
+2.0
+1.0
0.0
-1.0
-2.0
-3.0
-4.0
-5.0
-6.0
-7.0
-8.0
0
0.2
0.4
0.6
0.8
1


iteration step 7
$\cdot 10^{-6}$
+2.0
+1.6
+1.2
+0.8
+0.4
0.0
-0.4
-0.8
-1.2
-1.6
-2.0
0
0.2
0.4
0.6
0.8
1

### 4.2.2 Auswertungen

**Auswertung Iteration Step 1**

```
Input from job file "../Jobs/para74/Berechnung2/para74-2-step1_x":

A local extremum has been found at x=  +0.440588430408164:
           x                  err(x)
  ------------------     ------------------
  +0.440588430408164     -0.000020772361334
  +0.000000000000000     +0.000036813123467 (*)
  +1.000000000000000     -0.000012493107377

A zero has been found at   x=  +0.055464442769973
```

**Auswertung Iteration Step 7**

```
Input from job file "../Jobs/para74/para74-2-step7":

The exact local extrema of f-E(a) in interval I:
-----------------------------------------------
               x[i]                  y[i]
         ------------------   ------------------
i= 0:   +0.000000000000000   +0.000001830174958
i= 1:   +0.045496809970007   -0.000001851532562
i= 2:   +0.181066932667218   +0.000001830056835
i= 3:   +0.397876853010144   -0.000001831546150
i= 4:   +0.663792447729353   +0.000001831127068
i= 5:   +0.901319682783082   -0.000001831444282
i= 6:   +1.000000000000000   +0.000001829396744

Norm of error function:       0.000001851532562
Relative deviation:           0.011955403175477
```

```
The exact zeroes of f-E(a) in interval I:
-----------------------------------------
               x[i]
        ------------------
i= 0:   +0.011294676617833
i= 1:   +0.102473795624775
i= 2:   +0.280446199212420
i= 3:   +0.528325191814231
i= 4:   +0.792877548978730
i= 5:   +0.974238597882620
```

## 4.3 Zusammenfassung, Vergleich

Tabelle 4-1 enthält für diese Berechnungen (Spalte 1)

- die für die Berechnung der "best approximation" erforderliche Zahl von Iterationsschritten
- die Norm der Fehlerfunktion von "iteration step 1"
- die Norm der Fehlerfunktion der "best approximation"
- relative Abweichung der Alternante der "best approximation".

| | $\delta_{\mathbf{R}}$ | $\delta_{\mathbf{B}}$ | **itera-tions** | **norm of error function** iteration step 1 | **norm of error function** best approximation | **relative deviation** |
|---|---|---|---|---|---|---|
| 1 | 0.01 | 0.01 | 19 | 0.000010455220906 | 0.000001851393784 | 0.011463 |
| 2 | 0.01 | 0.01 | 7 | 0.000036813123467 | 0.000001851532562 | 0.011955 |

**Tabelle 4-1**

Die Norm der Fehlerfunktion der Startfunktionen:

- Berechnung 1: $n_1 = 0.0000107$ (s. [8], Abschnitt 1.1.2)
- Berechnung 2: $n_2 = 0.0000484$ (s. [8], Abschnitt 1.2.2)

Es gilt also

$$4 \times n_1 < n_2$$

Dennoch benötigt man für ein vergleichbares Ergebnis in Berechnung 1 mehr als doppelt soviele Iterationsschritte als in Berechnung 2.[6]

[6] vgl. entsprechenden Hinweis zu §7.4 in [1]

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